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Chapter ⇒ 2

ELECTROSTATIC POTENTIAL AND CAPACITANCE

Electrostatic Potential

"The electrostatic potential at a point in an electric field may be defined as the amount of the work done in bringing a unit +ve test charge from infinity to that point against the electrostatic force due to that field."

In bringing a charge q_0 from infinity to any point within the electric field work done is W.

∴ Electrostatic Potential at that point

$$V = \frac{\text{work done}}{\text{charge}}$$

$$\boxed{V = \frac{W}{q_0}}$$

It is a scalar quantity.

Unit ⇒ J/C or Volt (V)

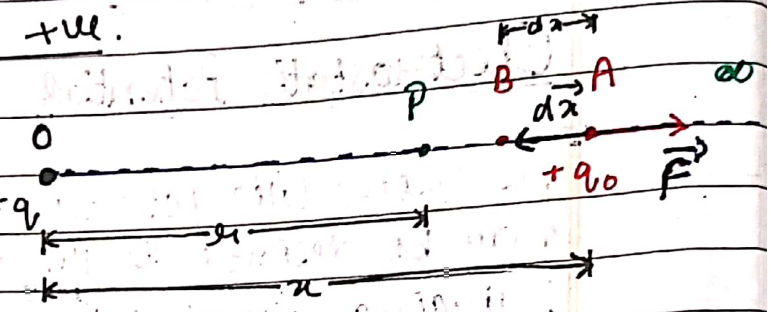
$$1 \text{ Volt} = \frac{1 \text{ Joule}}{1 \text{ coulomb}}$$

Thus, Electrostatic potential at a point in an electric field is said to be 1 volt if 1 Joule of work is done in moving a charge of 1 coulomb from infinity to that point against the electrostatic force due to that electric field.

The SI unit of Volt (V) has been named after Alessandro Volta.

ELECTROSTATIC POTENTIAL DUE TO A POINT CHARGE

(a) <u>when Source charge is +ve.</u>



Consider +q is a source charge kept at O.
Let P be any point at a distance 'r' from source charge where electrostatic potential is to be determined.

In order to find EP at P, let a charge $+q_0$ is brought from infinity to P. Let at any instant the test charge q_0 is at 'A' at a distance x from O, then force on $+q_0$ due to +q,

$$F = k.\frac{q q_0}{x^2} \quad \text{(along OA)} \longrightarrow (1)$$

Small work done to move the test charge from A to B by small distance dx

$$dW = \vec{F} \cdot \vec{dx}$$

$$dW = F\,dx \cos 180^\circ$$

$$dW = -F\,dx \longrightarrow (2)$$

Total work done to move the charge from '∞' to 'P'.

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} -F\,dx$$

$$W = -\int_{\infty}^{r} \frac{k q q_0}{x^2} . dx$$

$$W = -k q q_0 \int_{\infty}^{r} \frac{1}{x^2} . dx$$

$$W = -k q q_0 \left[\frac{-1}{x}\right]_{\infty}^{r}$$

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$$\therefore W = kqq_0\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\therefore W = \frac{kqq_0}{r} \quad \text{--- (3)}$$

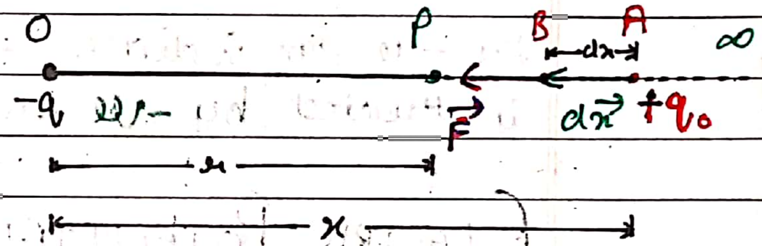
Now acc. to definition $V = \frac{W}{q_0}$ --- (4)

So from eq^n (3) & (4),

$$\boxed{V = k\frac{q}{r}} \quad \text{or} \quad \boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \quad \text{--- (5)}$$

(b) when source charge is −ve.

Consider a source charge $-q$ is kept at O.
Ref to fig.



When the test charge $+q_0$ is at A, then force on $+q_0$ due to $+q$ is

$$F = k\frac{qq_0}{x^2} \quad (\text{along AO}) \quad \text{--- (1)}$$

Small work done to move the test charge from A to B by small distance dx.

$$dW = \vec{F}\cdot\vec{dx}$$

$$dW = F\,dx\cos 0°$$

$$dW = F\,dx$$

Total work done to move the charge from '∞' to 'P'.

$$W = \int_{\infty}^{r} dW = \int_{\infty}^{r} F\,dx$$

$$W = \int_{\infty}^{r} \frac{kqq_0}{x^2}\cdot dx$$

$$W = \frac{kqq_0}{~~4~~}\left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$W = kqq_0\left[\frac{1}{\infty} - \frac{1}{r}\right]$$

$$W = -\frac{kqq_0}{r} \quad \text{--- (2)}$$

Now, $V = \frac{W}{q_0}$ --- (3)

∴ From (1) & (3) we get

$$\boxed{V = -\frac{kq}{r}} \qquad \boxed{V = -\frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$

The -ve sign indicates that the positive test charge (q_0) is attracted by -ve source charge.

ELECTRIC POTENTIAL DIFFERENCE

"The electric potential difference b/w two points in an electric field may be defined as the amount of the work done per unit +ve test charge in moving it from one point to another against the electrostatic force due to that field."

∴ Electric Potential Difference b/w two points A and B

$$V_{AB} = \frac{W_{AB}}{q_0}$$

SI unit ⇒ JC^{-1} or Volt

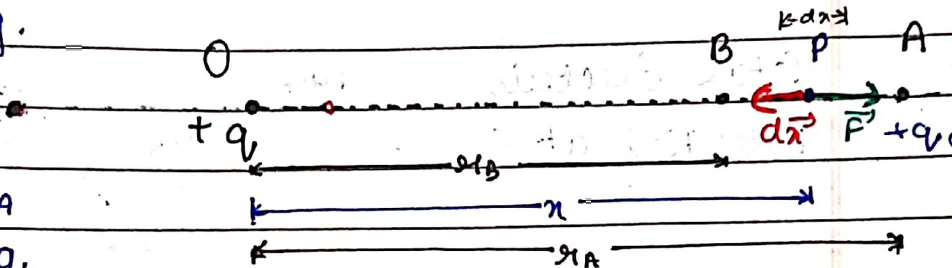
Dimensions ⇒ $[ML^2T^{-3}A^{-1}]$

NOTE :- (1) Test charge should be very small, so that it does not disturb the distribution of source charge.

(2) External force should be applied such that it does not produce any acceleration on test charge.

Expression for Electric Potential Difference.

Refer to fig.



$$V_A = k\frac{q}{r_A}$$

$$V_B = k\frac{q}{r_B}$$

Potential Difference b/w A and B

$$V_{AB} = V_A - V_B$$

$$\boxed{V_{AB} = kq\left[\frac{1}{r_A} - \frac{1}{r_B}\right]}$$

=) If q is +ve, then $V_B > V_A$

=) If q is -ve, then $V_B < V_A$

Q=) Calculate the electric potential at the surface of a gold nucleus. Given that the radius of nucleus is 6.6×10^{-15} m.

Solⁿ=) For gold $Z = 79$ $\quad q = Ze = 79e$

$$q = 79 \times 1.6 \times 10^{-19} \text{ C}.$$

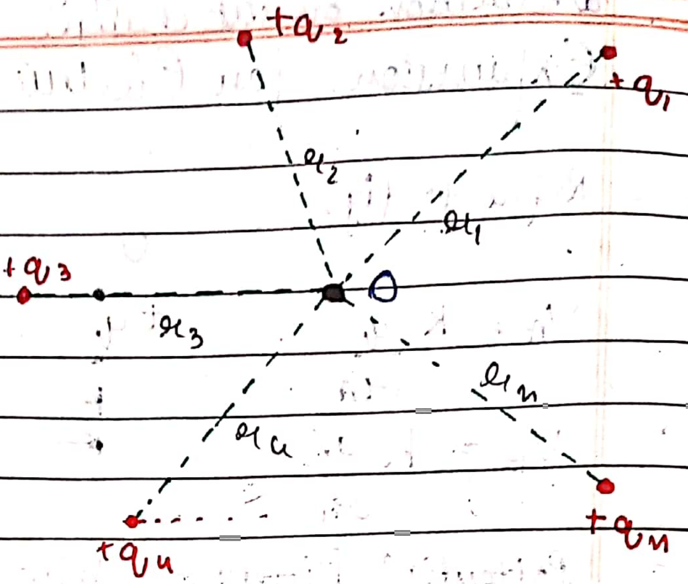
$$V = \frac{kq}{r} = \frac{9 \times 10^{9} \times 79 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-15}}$$

$$V = 1.723 \times 10^{7} \text{ V}$$

ELECTRIC POTENTIAL DUE TO GROUP OF CHARGES

As electric potential is a scalar quantity, therefore the total electric potential at a point due to a group of the charges is equal to the algebraic sum of the electric potentials due to each of the individual charge at that point.

Ref to fig.



Total Electric Potential at O

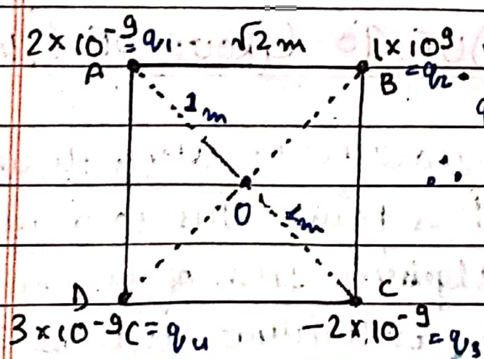
$$V_{total} = V_1 + V_2 + V_3 + \text{-----------} + V_n$$

$$V_{tot} = K\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \text{---------} \frac{q_n}{r_n}\right]$$

$$\boxed{V_{tot} = k\sum_{i=1}^{n} \frac{q_i}{r_i}}$$

NOTE ⇒ while adding the sign of the potential must be considered.

Q ⇒ Find the electric potential at the center of a square of side $\sqrt{2}m$ carrying the four charges $+2\times10^{-9}C$, $+1\times10^{-9}C$, $-2\times10^{-9}C$ and $+3\times10^{-9}C$.



The given situation is shown in fig. ∵ Electric potential is a scalar quantity.

$\therefore$ Σ V_{net} at $O = \frac{kq_1}{r} + \frac{kq_2}{r} + \frac{kq_3}{r} + \frac{kq_4}{r}$

$= \frac{k}{r}\left(2\times10^{-9} + 10^{9} + (-2\times10^{-9}) + 3\times10^{-9}\right)$

$= \frac{9\times10^{9}\times4\times10^{-9}}{1}$

$\boxed{V_{net} = 36V}$

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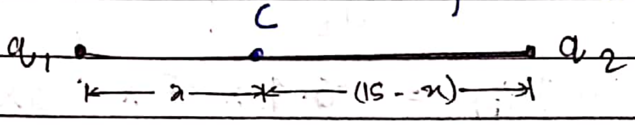
Q ⇒ Two charges of 3×10^{-8} C & 2×10^{-8} C are placed 15 cm apart. At what point on the line joining the two charges the EP will be zero? Take potential at infinity to be zero.

Sol^n ⇒ $q_1 = 3 \times 10^{-8}$ C & $q_2 = 2 \times 10^{-8}$ C.

Let EP be zero at 'C' at a distance x from q_1 as shown



as EP at C = 0

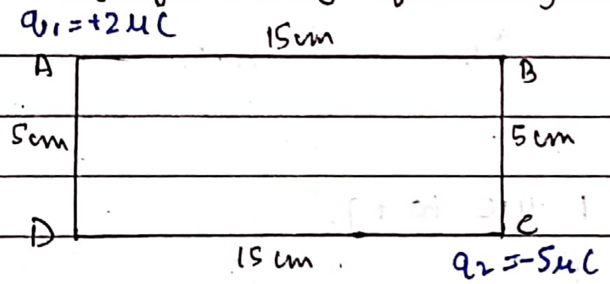
$\therefore V_{q_1} + V_{q_2} = 0$

$$\frac{kq_1}{x} + \left(-\frac{kq_2}{15-x}\right) = 0$$

$$\frac{q_1}{x} = \frac{q_2}{15-x} \Rightarrow \frac{3 \times 10^{-8}}{x} = \frac{2 \times 10^{-8}}{15-x}$$

$$45 - 3x = 2x \Rightarrow \boxed{x = 9\text{ cm}}$$

Q ⇒ The sides of a rectangle ABCD are 15 cm (AB = CD) and 5 cm (BC = DA). Two point charges of +2 μC and −5 μC are placed at the corners A and C respectively. Calculate the work done in carrying a charge of +3 μC from point B to D.



The given situation is shown in fig.

Total potential B $(V_B) = 9 \times 10^9 \left(\frac{2 \times 10^{-6}}{15 \times 10^{-2}} - \frac{5 \times 10^{-6}}{5 \times 10^{-2}}\right)$

$$= 9 \times 10^9 \times 10^{-4} \left(\frac{2}{15} - 1\right)$$

$$= \cancel{9}^{3} \times 10^5 \times -\frac{13}{\cancel{15}_{5}}$$

$$V_B = -7.8 \times 10^5\ V$$

Total potential at D $(V_D) = 9 \times 10^9 \left(\frac{2 \times 10^{-6}}{5 \times 10^{-2}} - \frac{5 \times 10^{-6}}{15 \times 10^{-2}}\right)$

$$= 9 \times 10^9 \times 10^{-4} \left(\frac{2}{5} - \frac{1}{3}\right)$$

$$V_D = \cancel{9}^{3} \times 10^5 \times \frac{1}{\cancel{15}_{5}} = 0.6 \times 10^5\ V$$

$$V_{BD} = V_B - V_D$$
$$V_{BD} = -7.8 \times 10^{5}\,V - 0.6 \times 10^{5}\,V$$
$$V_{BD} = -8.4 \times 10^{5}\,V$$

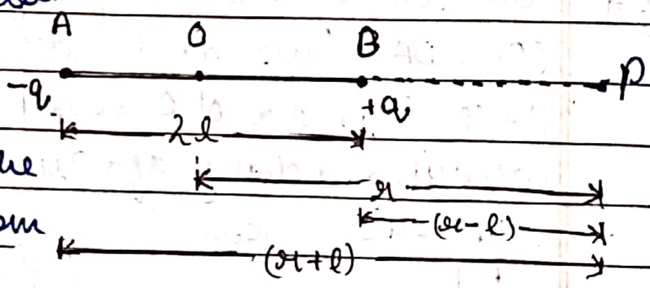
$$V_{BD} = \frac{W_{BD}}{q_0} \Rightarrow W_{BD} = V_{BD} \times q_0$$

$$W_{BD} = -8.4 \times 10^{5} \times 3 \times 10^{-6}$$
$$= \cancel{+19.6 \times 10^{-1}}\ -25.2 \times 10^{-1}$$
$$\boxed{\cancel{W_{BD} = -0.96\,J}} = -2.52 \times 10^{-1}\,J.$$

ELECTRIC POTENTIAL DUE TO AN ELECTRIC DIPOLE

(A) At any axial point

As shown in fig. Consider
AB ⇒ an electric dipole
2l ⇒ Dipole length
P ⇒ a point on the axis of the dipole at a distance r from its centre.



Electric Potential at P due to +q

$$V_B = \frac{kq}{(r-l)} \quad \text{--- (1)}$$

Electric Potential at P due to −q

$$V_A = \frac{-kq}{(r+l)} \quad \text{--- (2)}$$

∴ Total Electric Potential at P

$$V_{total} = V_B + V_A$$
$$= kq\left[\frac{1}{(r-l)} - \frac{1}{(r+l)}\right]$$
$$= kq\left[\frac{\cancel{r}+l-\cancel{r}+l}{r^2-l^2}\right]$$

$$V_{tot} = \frac{kq2l}{r^2-l^2}$$

$$V_{tot} = k\frac{p}{r^2-l^2} \quad (\text{as } q2l = p)$$

$$\boxed{V_{axial} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2-l^2}} \quad (3)$$

Special case ⇒ If $l << r$ (Short dipole)

$\therefore r^2 - l^2 \approx r^2$

$\therefore$ From eqn (3), $\boxed{V_{axial} = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2}}$ (4)

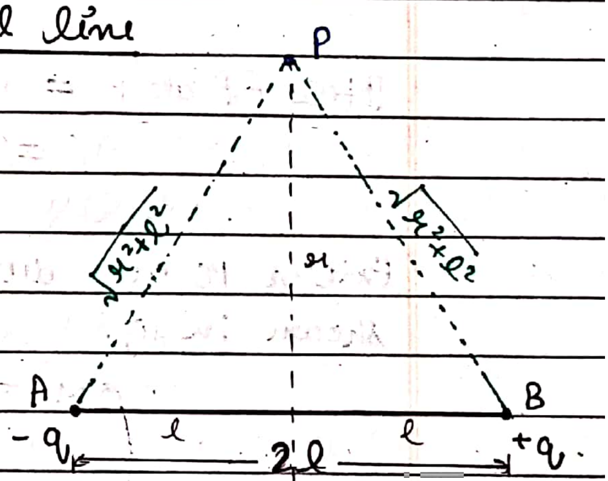
(B) At any point on its equitorial line

As shown in fig. consider

AB ⇒ an electric dipole

$2l$ ⇒ Dipole length

P ⇒ A point on the perpendicular bisector of the dipole at a distance r from its centre O.



EP at P due to +q

$$V_B = k\frac{q}{\sqrt{r^2+l^2}} \quad (1)$$

EP at P due to -q

$$V_A = -k\frac{q}{\sqrt{r^2+l^2}} \quad (2)$$

$\therefore$ Total EP at P

$$V_{eq} = V_A + V_B$$

$$\boxed{V_{eq} = 0}$$

NOTE:→ At any point on the equitorial line of dipole, the Electric Potential is zero but Electric Field is non-zero.

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(c) At any General Point

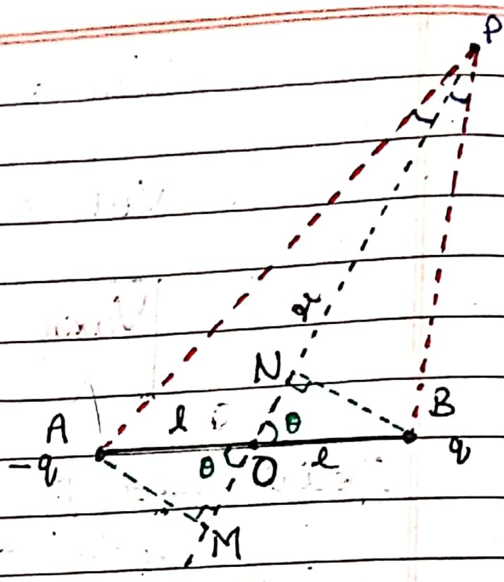
consider,

AB ⇒ an electric dipole

$2l$ ⇒ dipole length

P ⇒ any point where EP due to the dipole is to be found

r ⇒ distance of P from mid point of dipole. ∴ $OP = r$



EP at P due to $-q$: $V_A = -\dfrac{kq}{AP}$ — (1)

EP at P due to $+q$: $V_B = \dfrac{kq}{BP}$ — (2)

Total EP at P $= V = V_A + V_B$

$$V = kq\left[\frac{1}{BP} - \frac{1}{AP}\right] \quad \text{— (3)}$$

Extend PO and drop perpendiculars BN and AM on it as shown in fig.

$$OM = ON = l\cos\theta$$

$PM = PO + OM$

$PM = r + l\cos\theta$ — (4)

$PN = PO - ON$

$PN = r - l\cos\theta$ — (5)

If dipole is of short length then,

$AP \approx PM$ & $BP \approx PN$

then from eq^n (3)

$$V = kq\left[\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right]$$

$$V = kq\left[\frac{r + l\cos\theta - r + l\cos\theta}{r^2 - l^2\cos^2\theta}\right]$$

$$V = \frac{k(q\,2l)\cos\theta}{r^2 - l^2\cos^2\theta}$$

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$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{(r^2 - l^2\cos^2\theta)}} \quad (6)$$

⇒ Special Case :-

(i) when point P lies on axial line
then $\theta = 0^\circ$
∴ from eqⁿ (6)

$$V = \frac{1}{4\pi\epsilon_0}\frac{p\cos 0^\circ}{r^2 - l^2\cos^2 0^\circ}$$

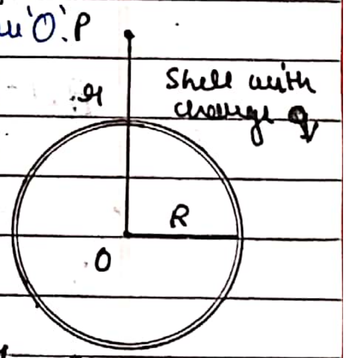
$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{p}{r^2 - l^2}}$$

(ii) when p lies on equitorial line
then $\theta = 90^\circ$
∴ from eqⁿ (6) $V = \frac{1}{4\pi\epsilon_0}\frac{p\cos 90^\circ}{r^2 - l^2\cos 90^\circ}$

$$\boxed{V = 0}$$

⇒ Electric Potential due to a uniformly Charged Thin Spherical Shell.

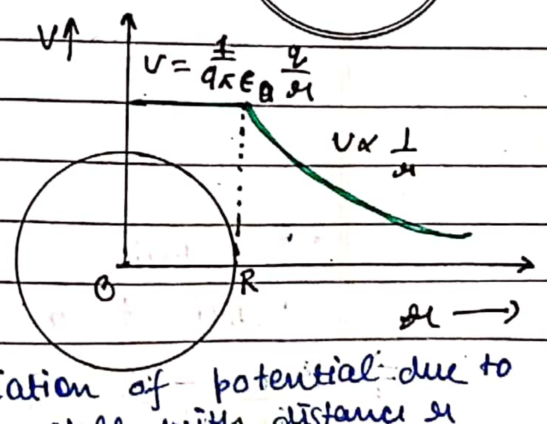
r ⇒ dist. of point 'P' from center 'O'. P
R ⇒ radius of shell
P ⇒ Point where EP due to the shell is to be found



(a) when point P lies outside the shell
(For $r > R$) $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$

(b) when point P lies on the surface of shel
(For $r = R$) $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

(c) when point P lies inside the shell
(For $r < R$) $V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$



(a) $V_{out} = \frac{\sigma R^2}{\epsilon_0 r}$ (as $q = 4\pi R^2\sigma$ ∴ $V_{out} = \frac{1}{4\pi\epsilon_0} \times \frac{4\pi R^2\sigma}{r}$)

(b) $V_{surf} = \frac{\sigma R}{\epsilon_0}$

Variation of potential due to charged shell with distance r from its centre.

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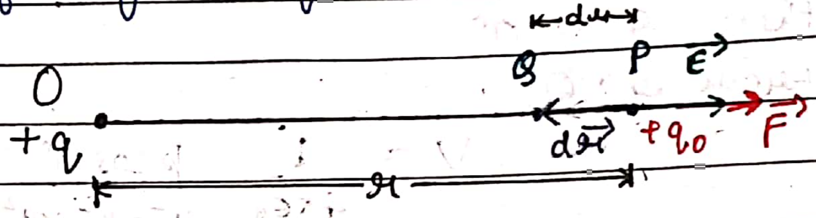
Relation Between Electric Field and Electric Potential

Let a point charge $+q$ is kept at O. Let P & Q are two points at a small distance dr apart as shown. Distance of P from $+q$ is r.



If V is the potential at P, then potential at Q will be $V + dV$

$\therefore$ Potential difference b/w P & Q $= dV$

If a charge $+q_0$ is placed at P then it will experience a force as shown in fig.

$$\vec{F} = q_0 \vec{E} \quad \text{--- (1)}$$

If q_0 is moved from P to Q then work done

$$W = \vec{F} \cdot \vec{dr}$$

$$W = F\, dr \cos 180^\circ$$

$$W = -(q_0 E)\, dr \quad \text{--- (2)}$$

Also,

$$W = \text{Potential difference} \times q_0$$

$$W = dV\, q_0 \quad \text{--- (3)}$$

$\therefore$ from (2) & (3)

$$-\cancel{q_0} E\, dr = dV \cancel{q_0}$$

$$\boxed{E = -\frac{dV}{dr}} \quad \text{--- (4)}$$

$\therefore$ "Electric field may also be defined as the –ve gradient of Electric Potential."

$\Rightarrow$ Here, –ve sign indicates that direction of $\vec{E}$ is in the direction of fall of potential.

Q⇒ The electric field $V_{(x)}$ in a region along x-axis varies with the distance x (in m) according to the relation $V_{(x)} = 4x^2$. Calculate the force experienced by a 1 μC charge placed at a point x = 1 m.

Solⁿ $V_x = 4x^2$ $q = 1\mu C = 10^{-6} C$

as $E = -\dfrac{dV}{dr}$ or $E = -\dfrac{dV}{dx}$

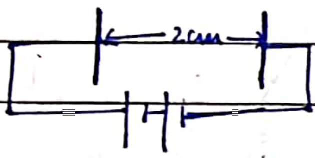
$E = -\dfrac{d(4x^2)}{dx} = -8x$

at $x = 1m$ $E = -8 \times 1 = -8 \; N/C$

using $F = qE$

$F = 10^{-6} \times (-8) = -8 \times 10^{-6} N$

Q⇒ Two metallic plates are 2cm apart and are connected to a battery of 1000 V. A proton is placed b/w them. Find out:-
(i) Electric field b/w the points
(ii) Force on the proton
(iii) will the proton experience different forces b/w the plates when placed at different places. Give reasons.



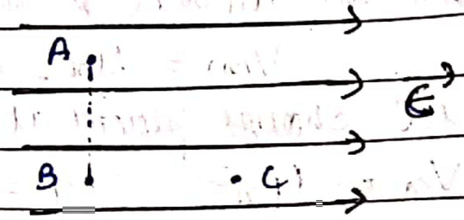
$dV = 1000 V$
$dr = 2cm = 0.02m$

(i) $E = -\dfrac{dV}{dr} = \dfrac{-1000}{0.02} = -50000 \; Vm^{-1}$

(ii) $F = qE = eE = 1.6 \times 10^{-19} \times 50{,}000$
$= 8 \times 10^{-15} N$

(iii) No because Electric field $\left(\dfrac{\sigma}{\epsilon_0}\right)$ b/w the plates will be constant (uniform) and hence the force on the proton will be uniform (constant).

Q ⇒ Which relation is correct for the given fig.



(a) $V_A = V_B = V_C$

(b) $V_A > V_B > V_C$

(c) $V_A = V_B < V_C$

✓(d) $V_A = V_B > V_C$

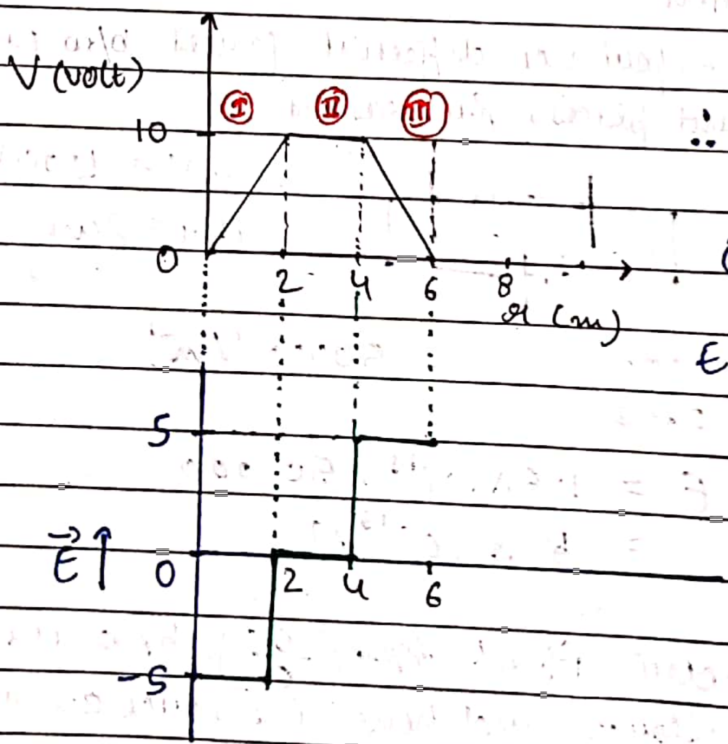
as A ∥ B ∴ $V_A = V_B$ and as $\vec{E}$ is in the direction of decrease in potential, so, $V_C < V_A = V_B$

NOTE :→ as $E = -\frac{dV}{dr}$

$E = -$ve gradient of Electric Potential

$$\boxed{E = -\text{ve slope of } V\text{-}r \text{ graph}}$$

Q⇒ Plot the E-r graph for the given V-r graph.



as $E = -$ slope of V-r graph

∴ $E_I = -\frac{dV}{dr} = -\frac{10}{2} = -5 V/m$

$E_{II} = 0$ (as $dV = 0$)

$E_{III} = -\frac{dV}{dr} = -\frac{(0-10)}{2} = 5 V/m$

NOTE (Additional Information) :-

(1) When we convert V function into E function then :-

(i) If there is only one variable

$$E = -\frac{dV}{dr}$$

⇒ (ii) If there are more variables :-

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$E_x = -\frac{\partial V}{\partial x}$	$E_y = -\frac{\partial V}{\partial y}$	$E_z = -\frac{\partial V}{\partial z}$
– (Partial derivative of V w.r.t x)	– (Partial derivative of V w.r.t y)	– (Partial derivative of V w.r.t z)

$$\therefore \boxed{\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right]}$$

Q ⇒ The electric potential in a region is represented as $V = 2x + 3y - z$. Obtain the expression for electric field strength.

Solⁿ ⇒

$$E = -\left[\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}\right]$$

$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x + \underbrace{3y - z}_{\text{constant}}) = 2$$

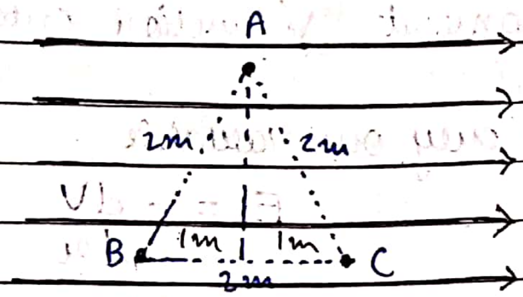
$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x + 3y - z) = 3$$

$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial y}(2x + 3y - z) = -1$$

$$\vec{E} = -[2\hat{i} + 3\hat{j} - \hat{k}]$$

$$\boxed{\vec{E} = -2\hat{i} - 3\hat{j} + \hat{k}}$$

Q ⇒ In uniform electric field $E = 10 N/C$ as shown below. Find :- (i) $V_A - V_B$ (ii) $V_B - V_C$ (iii) $V_C - V_A$



Clearly $V_B > V_A > V_C$

using $E = \frac{dV}{dx}$ ⇒ $dV = E \times dx$

(i) $V_A - V_B = -E \times dx$

$V_A - V_B = -10 \times 1 = -10 V$

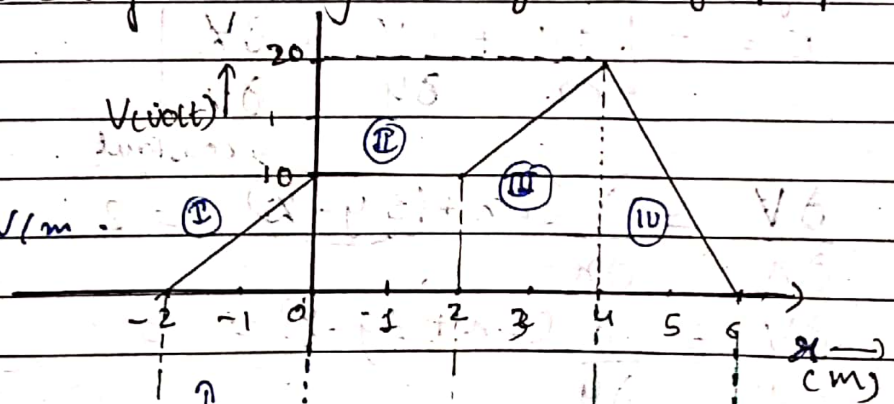
(ii) $V_B - V_C = E \times dx$

$= 10 \times 2 = 20 V$

(iii) $V_C - V_A = -E \times dx$

$= -10 \times 1 = -10 V$

Q ⇒ Corresponding to the following V-x graph plot E-x graph.



(I) $E_1 = -\frac{(10-0)}{0-(-2)}$

$E_1 = \frac{-10}{2} = -5 V/m$

(II) $E_2 = -\frac{(10-10)}{2-0}$

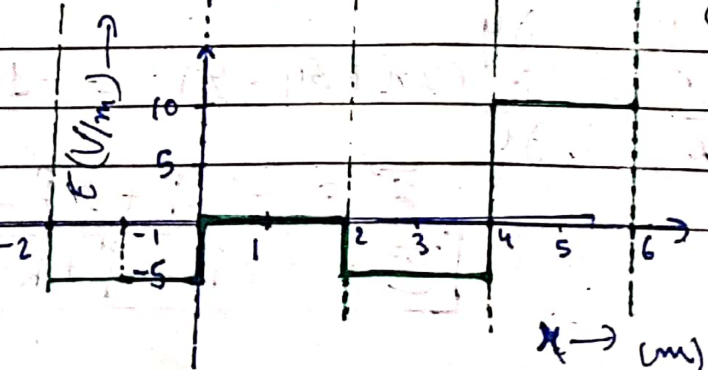
$E_2 = 0$

(III) $E_3 = -\frac{(20-10)}{4-2}$

$E_3 = -5$

(IV) $E_4 = \frac{-(0-20)}{6-4}$

$E_4 = \frac{20}{2} = 10$



Q3 ⇒ Find $\vec{E}$ if (i) $V = a(x^2 - y^2)$ (ii) $V = axy$.

(i) $V = ax^2 - ay^2$

$$\frac{\partial V}{\partial x} = \frac{\partial (ax^2 - ay^2)}{\partial x}$$

$$= 2ax$$

$$\frac{\partial V}{\partial y} = \frac{\partial (ax^2 - ay^2)}{\partial x}$$

$$= -2ay$$

$$\vec{E} = -[2ax\hat{i} - 2ay\hat{j}]$$

$$\vec{E} = -2ax\hat{i} + 2ay\hat{j}$$

(ii) $V = axy$

$$\frac{\partial V}{\partial x} = \frac{\partial (axy)}{\partial x} = ay$$

$$\frac{\partial V}{\partial y} = \frac{\partial (axy)}{\partial y} = ax$$

$$\vec{E} = -\{ay\hat{i} + ax\hat{j}\}$$

NOTE :→

Conversion of E function into V function

as $E = -\frac{dV}{dr}$ ∴ $dV = -E\,dr$

$$\boxed{dV = -\vec{E}\cdot d\vec{r}} \quad \text{——— (1)}$$

$$\boxed{\int dV = -\int \vec{E}\cdot d\vec{r}} \quad \text{——— (2)}$$

$$\int_A^B dV = \int_A^B \vec{E}\cdot d\vec{r}$$

$$\boxed{V_B - V_A = \int_A^B \vec{E}\cdot d\vec{r}} \quad \text{——— (3)}$$

where $d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

Q:⇒ Find $V_A - V_B$ (V_{AB}) in an electric field $\vec{E} = (2\hat{i} + 3\hat{j} + 4\hat{i})$ N/c where
$\vec{r}_A = (\hat{i} - 2\hat{j} + \hat{k})$ m $\vec{r}_B = (2\hat{i} + \hat{j} - 2\hat{k})$ m

Sol^n as $dV = -\vec{E} \cdot d\vec{r}$

$$V_{AB} = -\int_B^A \vec{E} \cdot d\vec{r}$$

$$V_{AB} = -\int_{(2,1,-2)}^{(1,-2,1)} (2\hat{i} + 3\hat{j} + 4\hat{k})(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

$$= \int_{(2,1,-2)}^{(1,-2,1)} (2dx + 3dy + 4dz)$$

$$= -\left[2x + 3y + 4z\right]_{(2,1,-2)}^{(1,-2,1)}$$

$$= -\left[(2\times1 + 3\times(-1) + 4\times1) - (4 + 3 - 8)\right]$$

$$= -[2 - 6 + 4 - 4 - 3 + 8]$$

$$V_{AB} = -[2 - 1] = -1V$$

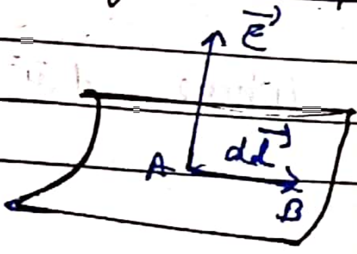
EQUIPOTENTIAL SURFACES

"Any surface, at every point on which the electric potential is same is called equipotential surface."

Properties :→

(1) Work done in moving a test charge, from one point to other over it is zero.

Proof ⇒ Ref to fig
If A & B are two points on the equipotential surfaces
then $V_A = V_B$



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$\therefore$ W_{AB} = (potential difference) $\times q$
$$W_{AB} = (V_A - V_B)\, q$$
$$W_{AB} = 0 \times q = 0$$

(2) Electric field is always perpendicular to the equipotential surface at every point.

Proof ⇒ as proved above

$$W_{AB} = 0$$
i.e. $$\vec{F} \cdot d\vec{l} = 0$$
$$q\vec{E} \cdot d\vec{l} = 0$$
$$\therefore \vec{E} \cdot d\vec{l} = 0 \quad [\text{as } q \neq 0]$$

as dot product of 2 vectors is zero when both are $\perp$ to each other, So,

$$\boxed{\vec{E} \perp d\vec{l}}$$

(3) No two equipotential surfaces can intersect each other.
This is because, if they do so, then at the points of intersection there will be two different directions of $\vec{E}$ which is not possible.

(4) Equipotential surfaces are closer together in the regions of strong field and farther apart in the regions of weak field for same change in potential i.e. potential difference remain constant

Proof ⇒ We know that $E = -\dfrac{dV}{dr}$ or $dr = -\dfrac{dV}{dE}$

For same change in the value of dV i.e., when dV = constant

$$dr \propto \frac{1}{E}$$

$\therefore$ If E is stronger, dr is small.
& if E is weaker, dr is larger.

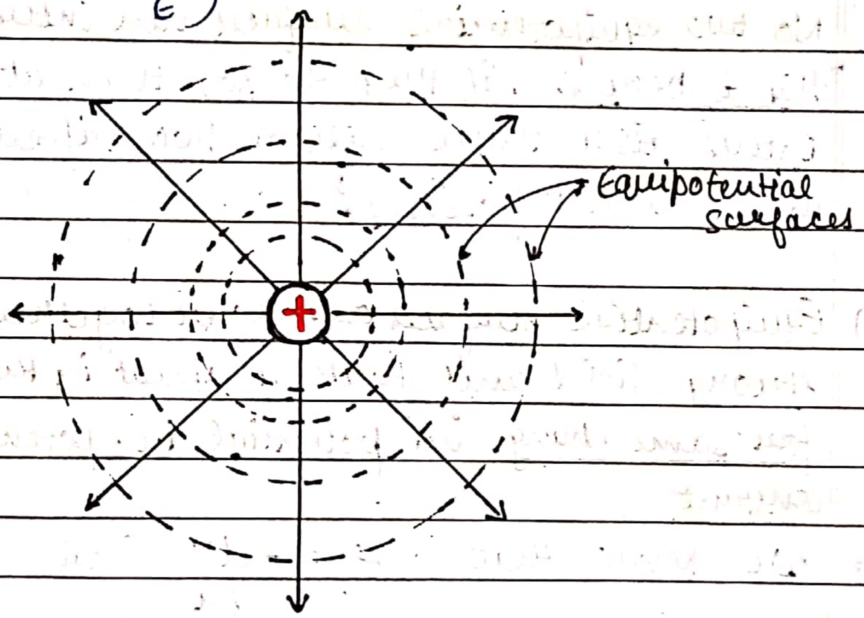
(5) The equipotential surfaces tell the direction of electric field

If a number of equipotential surfaces are drawn in an electric field for the same change in electric potential, then the direction of electric field is from the equipotential surfaces which are closer to each other to those which are more and more away from each other.

SHAPES OF EQUIPOTENTIAL SURFACES

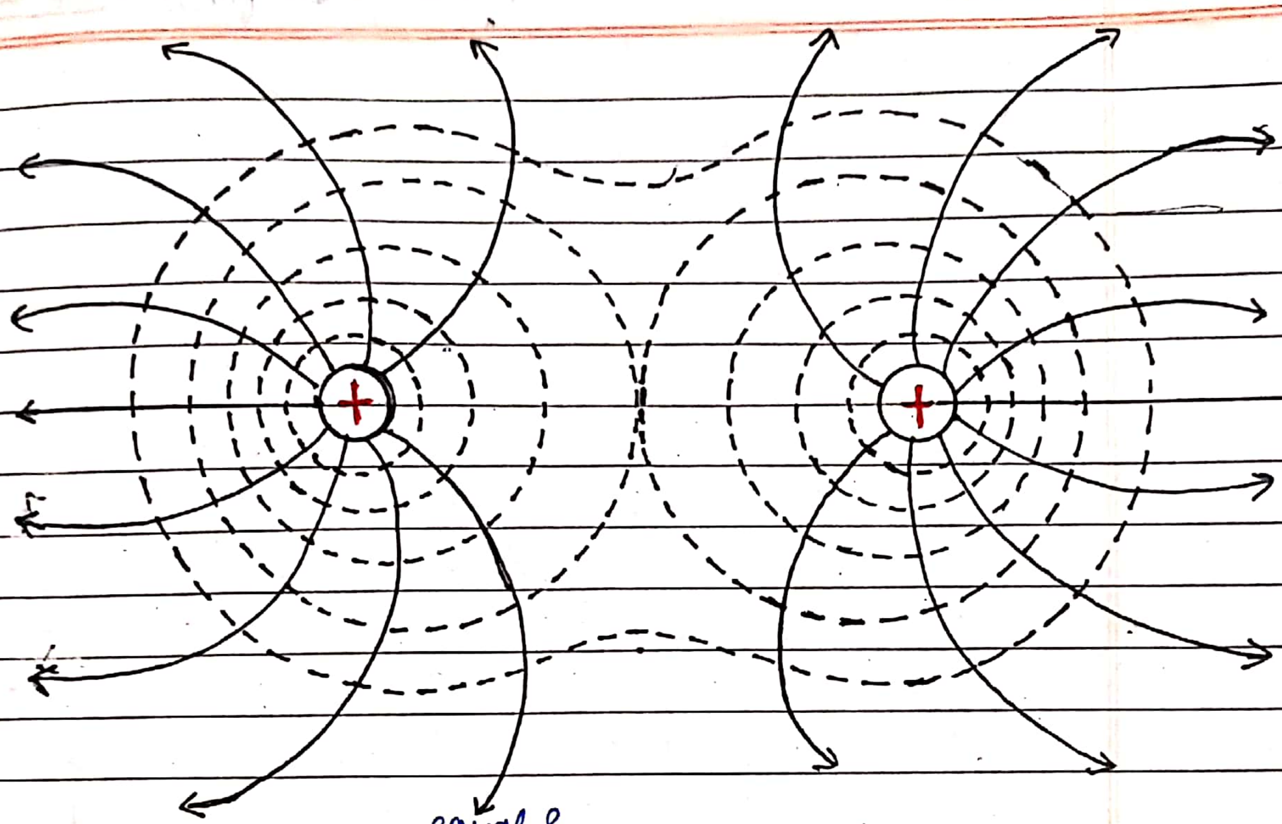
(A) For an isolated point charge:-

The equipotential surfaces will be cocentric spheres as shown. For same change in electric potential, the separation b/w surfaces keep increasing on moving away from the charge (as $d \propto \frac{1}{E}$).

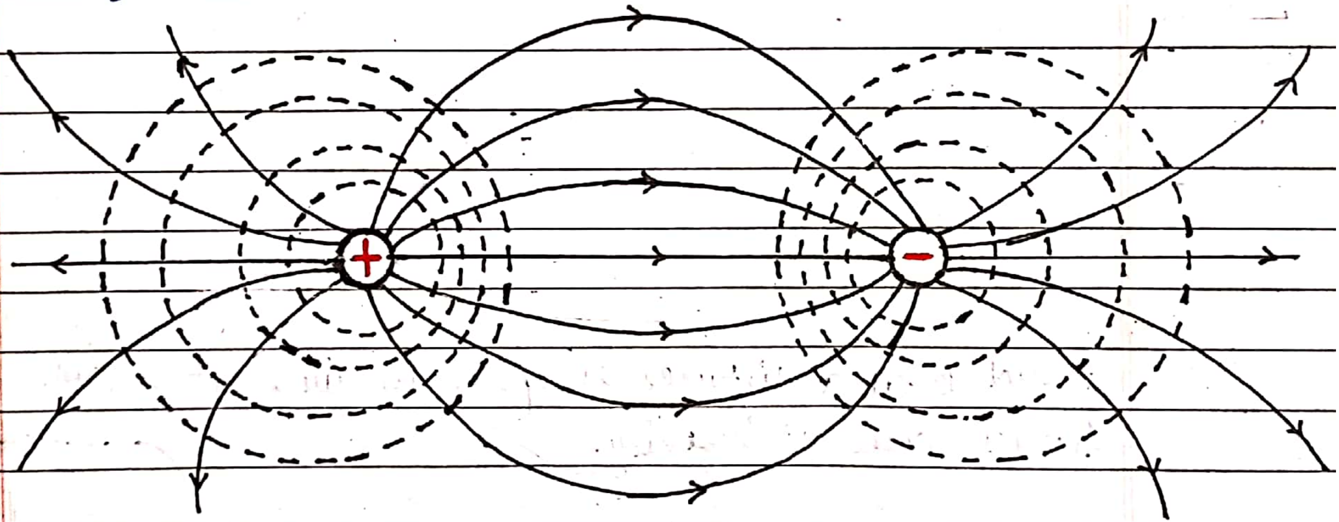


(B) For a system of two point charges:-

(1) When the charges are equal like:- The equipotential surfaces are far apart in the regions in b/w the two charges, indicating a weak field in such regions. → Ref. to fig.

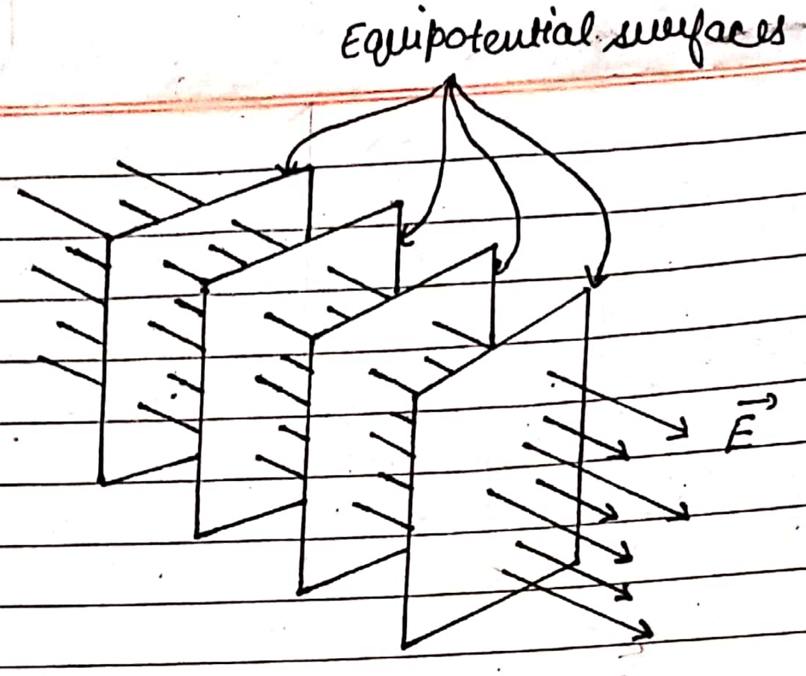


(2) When the charges are equal & unlike:- Ref to fig. The equipotential surfaces are close together in the region in b/w the two charges.



(C) For a uniform electric field:-

The equipotential surfaces in this case will be parallel, equidistant planes as shown in figure. As for same change in electric potential $dx \propto \frac{1}{E}$, therefore if E is constant then dx will also be constant.

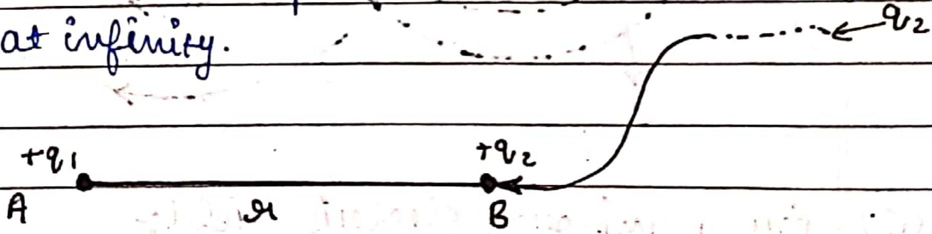


ELECTRIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

"The electric potential energy of a system of point charges may be defined as the amount of work done in bringing the charges constituting the system from infinity to their respective locations."

(A) Electric Potential Energy of system of two charges :-

Consider the two point charges q_1 and q_2 are placed at A and B at a distance r apart in air. Let initially the charges were at infinity.



Work done to bring the charge q_1 from infinity to A

$W_1 = 0$ ——— (1) (There is no external field against which work has to be done)

Work done to bring the charge q_2 from ∞ to B

$W_2 = (V)_{q_1} \times q_2$ $\left[\text{as } V = \frac{W}{q} \Rightarrow W = Vq\right]$

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$$w_2 = \frac{kq_1}{r} \times q_2 \quad \text{--- ②}$$

Total work done $W = w_1 + w_2 = 0 + \frac{kq_1q_2}{r}$

$$W = \frac{kq_1q_2}{r} = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}$$

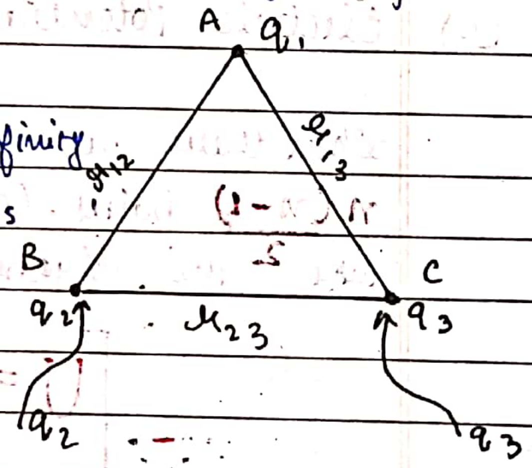
∴ E.P.E of system of two point charges = work done

So,

$$\boxed{U = \frac{1}{4\pi\epsilon_0} \frac{q_1q_2}{r}} \quad \text{--- ③}$$

(B) Potential energy of a system of three point charges.

As shown in fig. now we bring in the charge q_3 from infinity to point C.



W.D. in bringing charge q_1 from infinity to A: $W_1 = 0$ --- ① (as there is no charge to exert force on q_1) (external field)

W.D. in bringing the charge q_2 from ∞ to B

$$W_2 = (V_{q_1}) \times q_2$$

$$W_2 = \frac{kq_1 q_2}{r_{12}} \quad \text{--- ②}$$

When we bring the charge q_3 from infinity to C, the work has to be done against the forces exerted by q_1 and q_2.

$$\therefore W_3 = [(V_{q_1}) + (V_{q_2})] \times q_3$$

$$= \left(\frac{kq_1}{r_{13}} + \frac{kq_2}{r_{23}}\right) q_3$$

$$W_3 = K\left(\frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right) \quad \text{--- ③}$$

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$\therefore$ Total work done $W = W_1 + W_2 + W_3$

$$W = 0 + \frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}}$$

$$W = k\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]$$

$$W = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right] \quad (4)$$

EPE of the system = work done

$$\therefore \boxed{U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}}\right]} \quad (5)$$

(c) Electric Potential Energy of n point charges.

When there are n point charges then there will be total $\frac{n(n-1)}{2}$ pairs. Calculate the EPE of each pair and then take the algebraic sum.

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\sum_{\text{all pairs}} \frac{q_iq_j}{r_{ij}}}$$

$$\boxed{U = \frac{1}{2}\cdot\frac{1}{4\pi\epsilon_0}\sum_{\substack{i=1 \\ i\neq j}}^{N}\sum_{j=1}^{N}\frac{q_iq_j}{r_{ij}}}$$

As double summation counts every pair twice, to avoid this the factor $1/2$ has been introduced.

NOTE:- The electrostatic potential energy of a pair of like charges is +ve whereas for a pair of unlike charges it is -ve.

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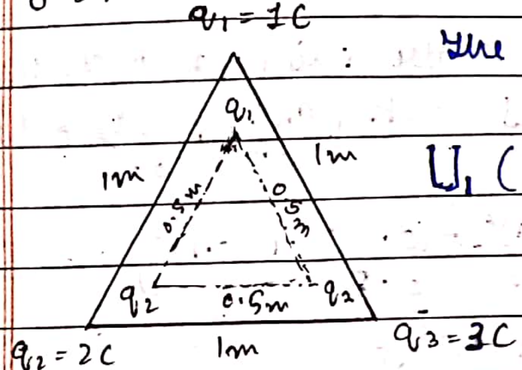
Units of ~~Pote~~ Electrostatic Potential Energy

Commonly used unit of EPE in atomic physics is electron volt (eV).

$$\boxed{1\,eV = 1.6 \times 10^{-9} J}$$

Q:) Three point charges 1C, 2C and 3C are placed at the corners of an equilateral triangle of side 1 m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle (having same centroid) of side 0.5 m.

Soln:) The given situation is shown in fig.



U_1 (EPE for initial configuration of charges)

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1q_2}{1} + \frac{q_2q_3}{1} + \frac{q_3q_1}{1}\right]$$

$$= \frac{1}{4\pi\epsilon_0} \left[(1\times2)+(2\times3)+(3\times1)\right]$$

$$U_1 = \frac{11}{4\pi\epsilon_0}$$

Similarly, U_2 (EPE of new configuration)

$$U_2 = \frac{1}{4\pi\epsilon_0}\left(\frac{(1\times2)}{0.5} + \frac{(2\times3)}{0.5} + \frac{(3\times1)}{0.5}\right)$$

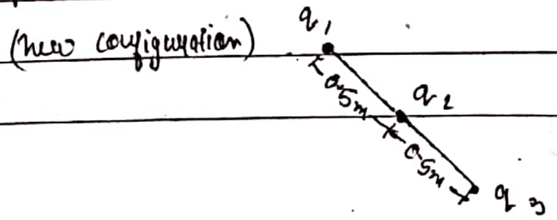
$$U_2 = \frac{11}{4\pi\epsilon_0} \times \frac{1}{0.5} = \frac{22}{4\pi\epsilon_0}$$

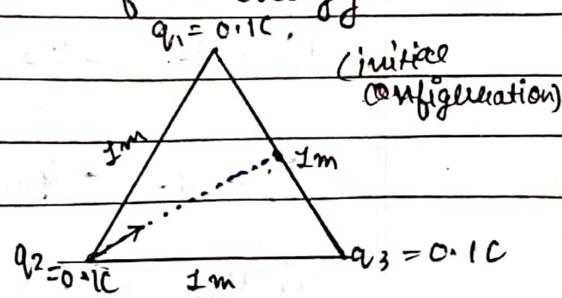
Work Done = Change in EPE

$$WD = \Delta U = U_2 - U_1 = \frac{1}{4\pi\epsilon_0}(22-11)$$

$$W = 9\times10^9 \times 11 \Rightarrow \boxed{W = 9.9 \times 10^{10} J}$$

Q:) Three point charges 0.1C each are placed at the corners of an equilateral triangle of side 1m. Calculate how many hours will be required to move one of the three charges on to the mid point of the line joining the other two if the energy is supplied at the rate of 1 kW.





Let the charges be q_1, q_2 & q_3 respectively.

EPE for initial configuration $U_1 = k\left[\frac{q_1q_2}{1} + \frac{q_2q_3}{1} + \frac{q_3q_1}{1}\right]$

$$U_1 = k\left[(0.1)^2 + (0.1)^2 + (0.1)^2\right]$$

$$U_1 = 3k \times 10^{-2}\ J$$

Let the charge q_2 is taken to the mid point of q_1 & q_3

EPE for new configuration $U_2 = k\left[\frac{q_1q_2}{0.5} + \frac{q_2q_3}{0.5} + \frac{q_3q_1}{1}\right]$

$$U_2 = k\left[\frac{(0.1)^2}{0.5} + \frac{(0.1)^2}{0.5} + (0.1)^2\right]$$

$$U_2 = 5k \times 10^{-2}\ J$$

~~Energy~~ Work done $= U_2 - U_1$

$$W = 2k \times 10^{-2}\ J$$

Power supplied $= 1kW = 1000W$

$$P = \frac{W}{t} \Rightarrow t = \frac{W}{P}$$

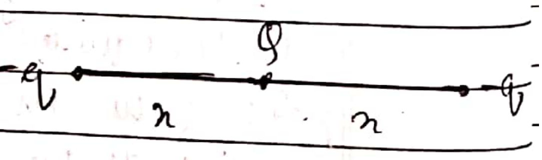
$$t = \frac{2 \times 9 \times 10^9 \times 10^{-2}}{1000}\ s$$

$$t = 18 \times 10^4\ s$$

$$t = \frac{18 \times 10^4}{3600}\ hr = \frac{1}{2} \times 10^2 = 50\ hrs.$$

Q ⇒ Three point charges $-q$, Q and $-q$, are placed at equal distances in a straight line. The Electrostatic potential energy of the system of the charges is 0. Find the ratio $\frac{q}{Q}$.

Sol^n EPE $U = k\left[\frac{(-qQ)}{x} + \frac{(-qQ)}{x} + \frac{(-q \times -q)}{2x}\right]$



$$U = k\left[\frac{-2qQ}{x} + \frac{q^2}{2x}\right]$$

$$U = k\left[\frac{-4qQ + q^2}{2x}\right]$$

Given $U = 0$

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$\Rightarrow K\left\{-\frac{4qQ}{2r}+q^2\right\}=0$

$\Rightarrow -4qQ+q^2=0$

$\Rightarrow 4Q=q$

$\Rightarrow \boxed{\frac{q}{Q}=4}$

Q⇒ Two electrons are moving towards each other with a velocity of 10^6 m/s. Upto what minimum distance they approach each other.

Sol^n Total kinetic energy of electrons $KE = 2\times\frac{1}{2}mv^2$
where m is the mass of electron & v is their velocity.
$(m = 9.1\times10^{-31}\,kg)$

Total potential energy of electrons $U = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$
where e → the charge on 1 electron
$= 1.6\times10^{-19}\,C$

When they approach closest distance (r) their total kinetic energy is converted into potential energy.

$$KE = U$$

$$\cancel{2}\times\frac{1}{\cancel{2}}mv^2 = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

$$r = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mv^2}$$

$$r = \frac{9\times10^9\times(1.6\times10^{-19})^2}{9.1\times10^{-31}\times10^{6}}$$

$$r = 2.53\times10^{-10}\,m$$

CONDUCTORS AND INSULATORS (or Dielectrics)

Conductors ⇒ Conductors are those materials through which electric charges can flow easily. For eg ⇒ metals, human body, earth, electrolytes, etc.

⇒ In metallic conductors, the free electrons (the electrons of the outer shells of the atoms which are loosely bound to the nucleus) are the charge carriers.

⇒ In electrolytic conductors both −ve & +ve ~~charges~~ ions are the charge carriers.

Insulators ⇒ Insulators are those materials through which electric charge can not flow. For eg ⇒ glass, rubber, plastic, paper, wax, mica, wood, etc.

Insulators are also called dielectrics.

Thus, "the dielectrics are the insulators which when placed in an electric field, induced charges appear on their surfaces."

⇒ In insulators, charges are bounded. Due to absence of free charges insulators are poor conductors of electricity.

Behaviour of a Conductor in an Electric Field

When placed in electrostatic fields, the conductors show the following properties :→

(1) When a conductor is placed in an electric field, then the charges appear on the surface of the conductors due to which an equal and opposite electric field is set up in the conductor and hence net electric field inside the conductor becomes zero.

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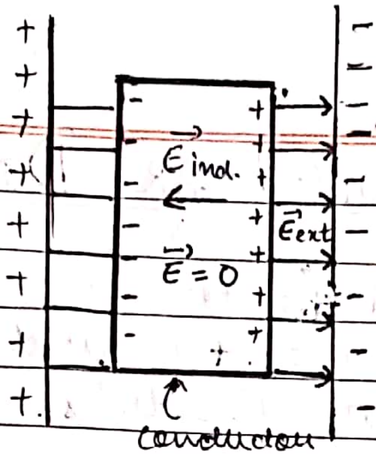
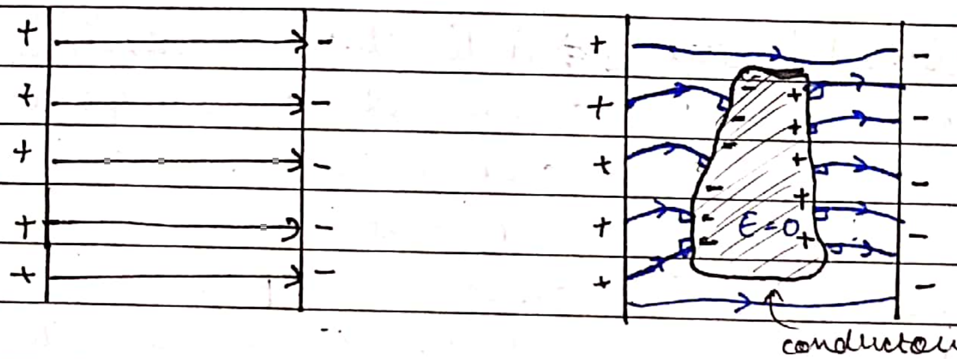


(2) when a conductor is placed in an electric field, the electric field is $\perp$ to the surface of the conductor.



(3) The charges resides only at the surface of the conductor and the net charge in the interior of a conductor is zero.

(4) The entire body of the conductor including its surface is at constant potential.

as inside the conductor electric field $(\vec{E}) = 0$

$$\therefore \frac{dV}{dr} = 0 \qquad \left[\text{as } E = \frac{dV}{dr} \text{ (in magnitude)}\right]$$

$$\frac{d}{dr}(V) = 0$$

$$\therefore \boxed{V = \text{constant}}$$

(5) The electric field at the surface of the charged conductor is $\boxed{E = \frac{\sigma}{\varepsilon_0}}$

(6) Electric field is zero in the cavity of a hollow charged conductor. The cavity inside the conductor remains shielded from outside electric influence. This is known as Electrostatic Shielding.

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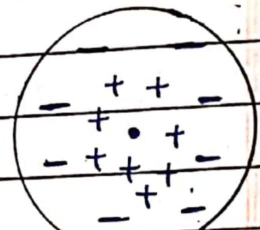
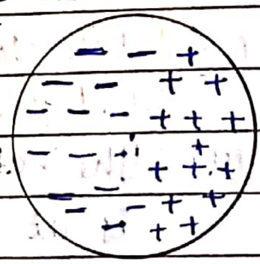
Polar and Non-Polar Dielectrics

Non Polar Dielectric ⇒ A molecule in which the centre of mass of +ve and −ve charges coincide is called a non polar molecule. The dielectrics made of non-polar molecules are called non-polar dielectrics.
For such dielectric molecules, dipole moment $\vec{p} = 0$ as there is zero separation b/w +ve and −ve charge.
eg ⇒ H_2, N_2, CO_2, O_2, CH_4, etc.



Non Polar

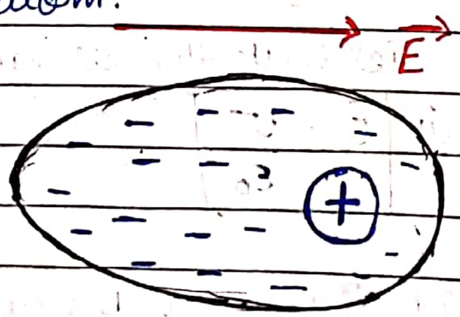
Polar Dielectric ⇒ A molecule in which the centre of mass of +ve charges & −ve charges does not coincide is called a polar molecule. The dielectrics made of polar molecules are called polar dielectrics.
In such dielectric molecules $\vec{p} \neq 0$.
eg ⇒ HCl, NH_3, CO, CH_3OH, H_2O, etc.



Polar

Polarization

"The stretching of the dielectric atoms due to the displacement of the charges in the atoms under the action of an applied electric field and hence developing a net dipole, is called polarization."
A non-polar dielectric atom hence converts into a polar dielectric atom.



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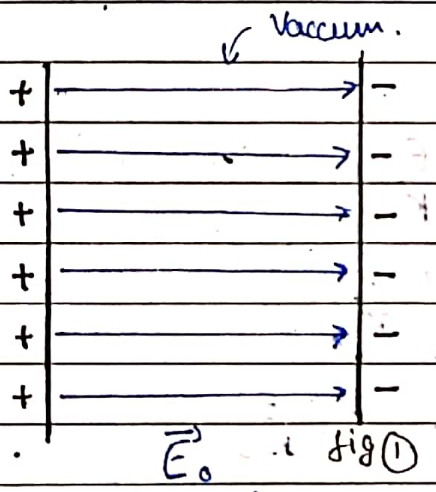
Polarisation of Dielectric Slab

Consider a rectangular dielectric slab of non-polar dielectric atoms is introduced into an external electric field $\vec{E_0}$. Each molecule of the dielectric gets polarised due to which the induced charges will appear on its surface as shown. These induced charges produce electric field $\vec{E_p}$ called electric field due to polarization. The two electric fields $\vec{E_0}$ and $\vec{E_p}$ are in opposite direction. Hence, the electric field within the dielectric becomes,

$$\boxed{E = E_0 - E_p} \quad \text{(in magnitude)}$$

E is called reduced value of electric field.



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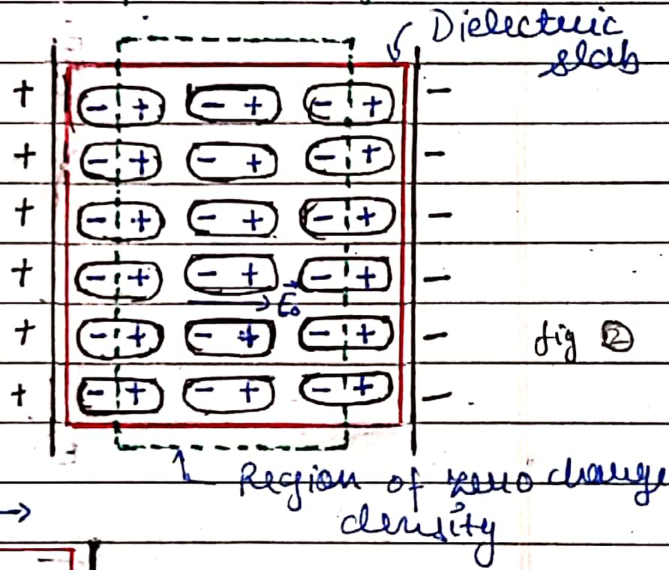

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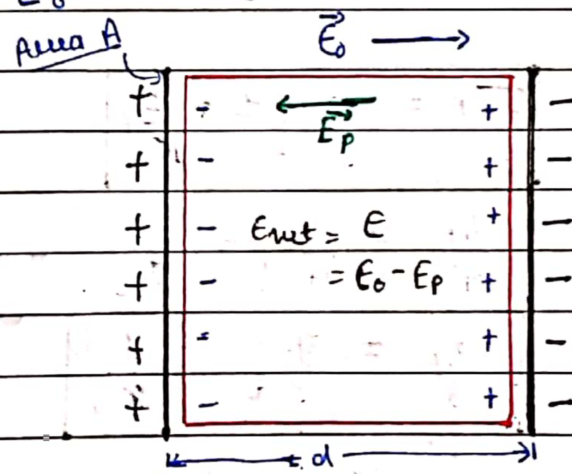

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⇒ **Dielectric Constant** ⇒ The dielectric constant of a dielectric medium may be defined as the ratio of the strength of the applied electric field to the strength of the reduced value of electric field when dielectric is placed in an external field.

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POLARISATION OF DIELECTRIC SLAB

Consider a rectangular dielectric slab of non-polar dielectric atoms is introduced into an external electric field $\vec{E_0}$. Each molecule of the dielectric gets polarised due to which the induced charges will appear on its surface as shown. These induced charges produce electric field $\vec{E_p}$ called electric field due to polarization. The two electric fields $\vec{E_0}$ and $\vec{E_p}$ are in opposite direction. Hence, the electric field within the dielectric becomes,

$$\boxed{E = E_0 - E_p} \quad \text{(in magnitude)}$$

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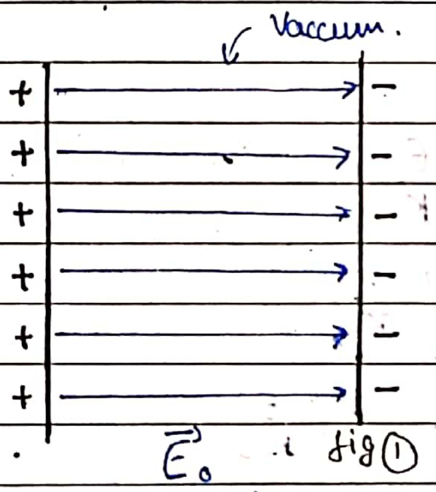

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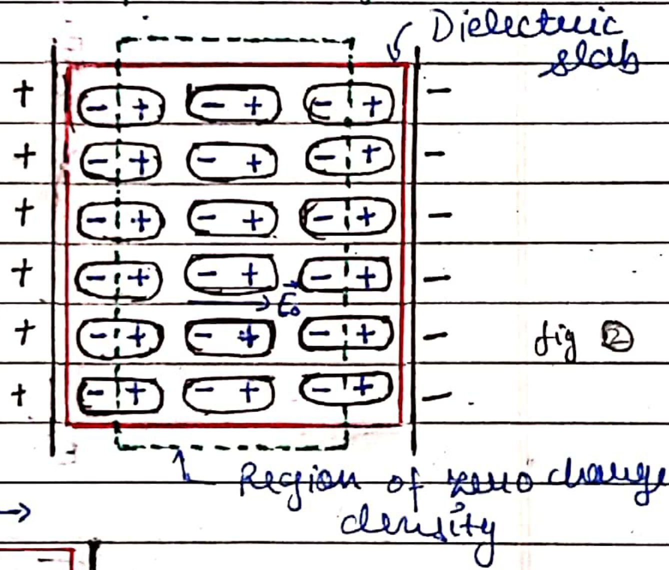

fig ②

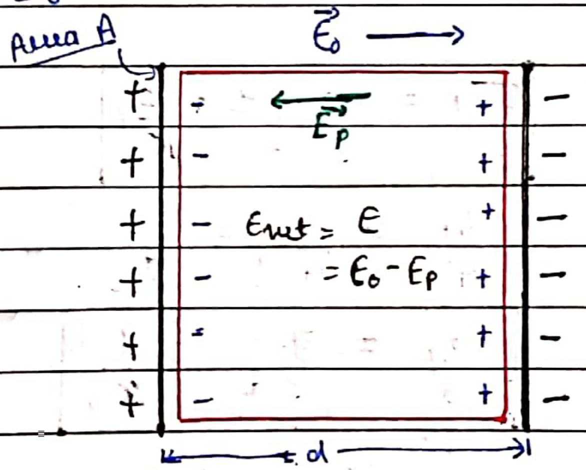

fig ③

⇒ **Dielectric Constant** ⇒ The dielectric constant of a dielectric medium may be defined as the ratio of the strength of the applied electric field to the strength of the reduced value of electric field when dielectric is placed in an external field.

Thus, dielectric constant (K) or relative permittivity (ϵ_r) is –

$$\boxed{K = \frac{E_0}{E} = \frac{E_0}{E_0 - E_p}}$$

NOTE :→

① Electric field is reduced when dielectric is placed in an external electric field.

② The induced charge in the dielectric can be calculated as –

$$E = E_0 - E_p \quad \text{——— ①}$$

$$K = \frac{E_0}{E} \Rightarrow E = \frac{E_0}{K} \quad \text{——— ②}$$

from ① & ②

$$\frac{E_0}{K} = E_0 - E_p$$

$$E_p = E_0 - \frac{E_0}{K}$$

$$E_p = E_0\left[1 - \frac{1}{K}\right]$$

$$\frac{\sigma_p}{\epsilon_0} = \frac{\sigma}{\epsilon_0}\left[1 - \frac{1}{K}\right]$$

$$\boxed{\sigma_p = \sigma\left[1 - \frac{1}{K}\right]} \quad \text{——— ③}$$

$$\frac{q_p}{A} = \frac{q}{A}\left[1 - \frac{1}{K}\right]$$

$$\boxed{q_p = q\left[1 - \frac{1}{K}\right]} \quad \text{——— ④}$$

induced charge

⇒ For dielectric $K > 1$, then $\boxed{q_p < q}$

⇒ For conductor $K = \infty$ $\boxed{q_p = q}$

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Polarisation Density

'The induced dipole moment developed per unit volume of a dielectric when placed in an external electric field is called polarisation density.'
It is denoted by 'P'.
If $p \Rightarrow$ induced electric dipole moment acquired by an atom
$N \Rightarrow$ no. of atoms per unit volume ~~then~~
Then, $\boxed{P = Np}$ — ①

If $A \Rightarrow$ area of each plate as shown in fig ③
$d \Rightarrow$ distance b/w the two plates.
$\therefore$ volume $= Ad$
If $+q_i$ & $-q_i$ are the induced charges then electric dipole moment $= q_i \times d$

Polarisation density = Induced electric dipole moment per unit volume

$$P = \frac{q_i \times \cancel{d}}{A\cancel{d}} = \frac{q_i}{A}$$ = charge induced per unit surface area.

$$\therefore \boxed{P = \sigma_p} \quad — ②$$

Electric Susceptibility

The polarisation density of a dielectric slab is directly proportional to the induced value of the electric field i.e.;
$\vec{P} \propto \vec{E}$ or $\boxed{\vec{P} = \epsilon_0 \chi \vec{E}}$ — ③
χ — electric Susceptibility

Clearly;
$$\boxed{\chi = \frac{\vec{P}}{\epsilon_0 \vec{E}}}$$
Thus, the ratio of the polarisation density to ϵ_0 times the electric field is called the electric susceptibility of the dielectric

$\Rightarrow$ χ is a dimensionless quantity.

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Relation b/w χ & K

as $E = E_0 - E_P$

$$E = E_0 - \frac{\sigma_P}{\epsilon_0}$$

$$E = E_0 - \frac{P}{\epsilon_0} \quad (\text{from } ② \; P = \sigma_P)$$

$$E = E_0 - \frac{\cancel{\epsilon_0} \chi E}{\cancel{\epsilon_0}} \quad (\text{from } ③ \; P = \epsilon_0 \chi E)$$

$$E + \chi E = E_0$$

$$E(1+\chi) = E_0$$

$$1 + \chi = \frac{E_0}{E}$$

$$\therefore \boxed{1 + \chi = K} \; — ④$$

DIELECTRIC STRENGTH

The molecules of a dielectric undergo more and more stretching if the strength of the applied electric field is increased gradually. At one stage electrons break up from the molecules of the dielectric. Due to this electric breakdown the dielectric becomes conducting.

"The maximum amount of the electric field that can be applied to the dielectric without its electric breakdown is called its dielectric strength."

SI unit ⇒ Vm^{-1}

Practical unit ⇒ kV/mm

eg ⇒ Dielectric strength of some media in kV/mm :-

1) Vaccum ⇒ ∞
2) Air ⇒ 0.8
3) Porcelain ⇒ 4
4) Mica ⇒ 160
5) Pyrex Glass ⇒ 13
6) Paper ⇒ 14
7) Water ⇒ –

Electrical Capacitance

'It is the ability of a conductor to store electric charge.'

When an insulated conductor is given some charge, it acquires a certain potential. If we increase the charge on a conductor its potential also increases.

$\therefore$ Charge $(q) \propto$ potential (V)

$$\boxed{q = CV} \quad \text{— (1)}$$

C — a constant called electrical capacitance.

$\therefore$ from (1),

$$\boxed{C = \frac{q}{V}} \quad \text{— (2)}$$

Thus, the electrical capacitance of a conductor may be defined as the ratio of the charge supplied to the potential raised.

from eq^n (2) if $V = 1$, then $\boxed{C = q}$

Thus, electrical capacitance of a conductor is numerically equal to the electric charge required to raise its potential by one volt.

from eq^n (2) SI unit of $C \Rightarrow \frac{\text{Coulomb}}{\text{volt}} = CV^{-1}$

$= \text{Farad } (F)$

1 Farad :→ as $C = \frac{q}{V}$ $\therefore 1\,F = \frac{1\,C}{1\,V}$

$\therefore$ 'The capacitance of a conductor is said to be 1 Farad if on supplying a charge of 1 C to its, its potential is raised to 1 V.'

Dimensions :- $\frac{A^2T^2}{MLT^{-2}L} = [M^{-1}L^{-2}T^4A^2]$

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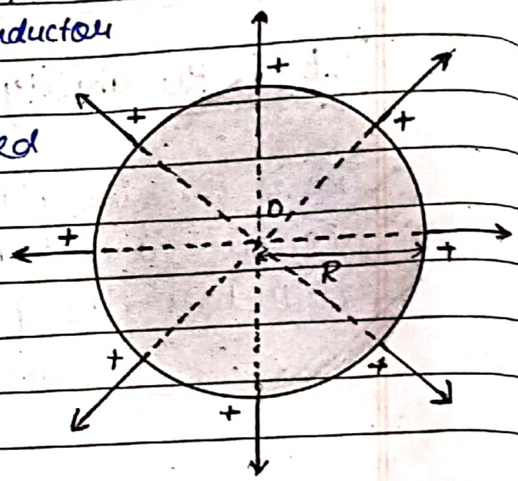
NOTE :→ Capacitance depends on following factors :→
(i) Size & shape of the conductor.
(ii) Nature (permittivity) of the surrounding medium.
(iii) Presence of other conductors near it.

Capacitance of an Isolated Spherical Capacitor

Consider an isolated spherical conductor of radius R.
+q → charge uniformly distributed over its entire surface.



The potential of the conductor

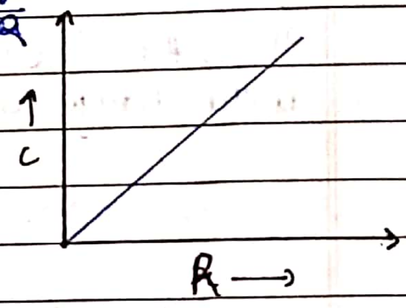
$$V = \frac{kq}{R} \quad \text{—} \ ①$$

as $$C = \frac{q}{V} \quad \text{—} \ ②$$

∴ from ① & ② $$C = \frac{q}{\frac{kq}{R}} = \frac{R}{k}$$

$$\therefore \boxed{C = 4\pi\epsilon_0 R} \ \text{—} \ ③$$

clearly $$\boxed{C \propto R}$$



⇒ **NOTE :→**

as $C = 4\pi\epsilon_0 R$

$$\epsilon_0 = \frac{C}{4\pi R}$$

another SI unit of $\boxed{\epsilon_0 = F\,m^{-1}}$

Capacitance of Earth

as earth is a spherical conductor of radius 6400 km

$$\therefore r = 6400 \times 10^3 \, m$$

for spherical conductor $C = 4\pi\epsilon_0 R$

$$C = \frac{1}{9\times10^9} \times 6.4 \times 10^6 = \frac{6.4 \times 10^{-3}}{9}$$

$$C = 0.711 \times 10^{-3} F = 711 \times 10^{-6} F$$

$$\boxed{C = 711\,\mu F}$$

This shows that 1F is very large unit of capacitance.

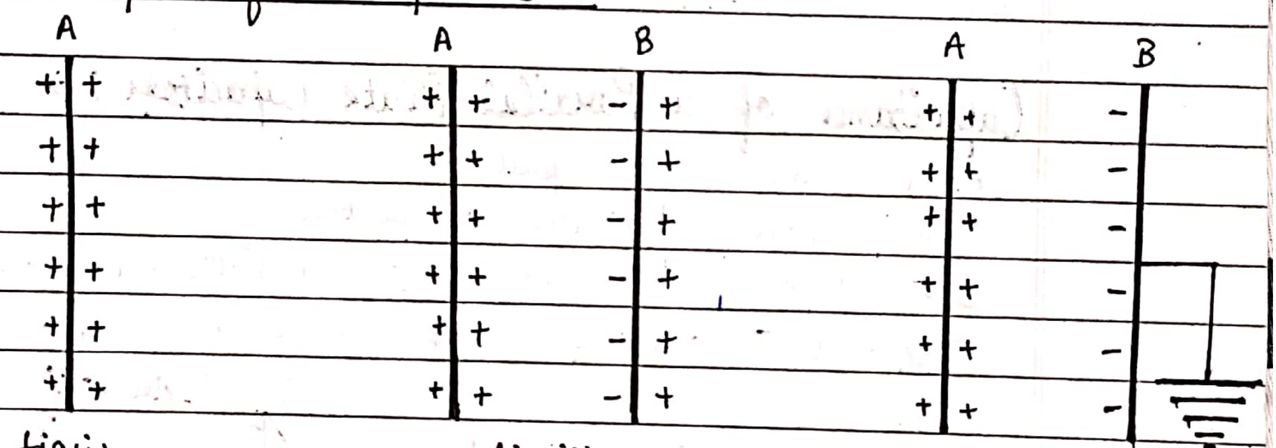
CAPACITOR

An arrangement of 2 conductors separated by an insulating medium which is used to store electric charge & electric energy.

"It is a device used to store a large amount of charge on it."

A conductor can store a small amount of charge and hence possesses a small capacitance. If some how we decrease the potential of the conductor, then in order to bring it again on the same potential, more charge can be given to it and thus the capacitance of the conductor will be increased.

Principle of a Capacitor



fig(i) fig(ii) fig(iii)

Consider the plate A is charged to its maximum potential. If a similar uncharged plate B is placed near it then the charges are induced on plate B as shown. The negative charge on plate B tends to reduce where as the +ve charge on plate B tends to increase potential of plate A. As −ve charge is more closer, potential of plate A is lowered by a small amount. Now, if plate B is connected to the ground as shown, +ve charge on it will disappear and only the −ve charge will remain on it. Thus, the potential of plate A is lowered by a large amount. To bring the potential of plate A again to its initial value, a large amount of charge has to be supplied to it. Thus, the capacitance of plate A is increased.

Thus, 'A capacitor is an arrangement of two metallic conductors so that if one is connected to the ground, the other has an ability to store a large amount of charge on it.'

Parallel Plate Capacitor

It is the most commonly used capacitor. It consists of two conducting plates parallel to each other, separated by a small distance.

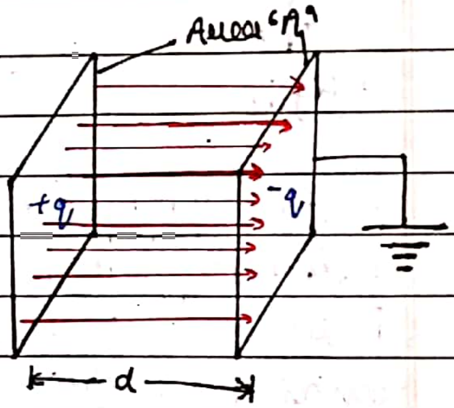
Capacitance of a Parallel Plate Capacitor :→

Let A = area of each plate

d = distance b/w the two plates.

$\pm\sigma$ = uniform surface charge densities on the two plates.

$\pm q = \pm\sigma A$ = total charge on each plate.



In the outer regions, the electric fields due to the two charged plates cancel out. The net field is 0.

$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

In the inner region, the net electric field is

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \quad \text{—— (1)}$$

Potential difference b/w the plates

$$V = Ed \quad \text{—— (2)} \quad \left(\text{as } E = \frac{V}{d}\right)$$

from ① & ②

$$V = \frac{\sigma}{\epsilon_0} d = \frac{q}{A\epsilon_0} d \quad —③ \quad \left(as\ \sigma = \frac{q}{A}\right)$$

Now, as $C = \frac{q}{V}$ — ④

∴ from ③ & ④ $\quad C = \dfrac{q}{\dfrac{q\,d}{A\epsilon_0}}$

$$\boxed{C = \frac{\epsilon_0 A}{d}} \quad —⑤$$

Thus, the factors on which the capacitance of a parallel plate capacitor depends are :-

(1) Area of the plate $(C \propto A)$
(2) Distance b/w the plates $(C \propto \frac{1}{d})$
(3) Permittivity of the medium b/w the plates $(C \propto \epsilon_0)$.

⇛ NOTE:→

If the medium b/w the plates is dielectric of absolute permittivity ϵ, then the capacitance is given by :-

$$C' = \frac{\epsilon A}{d} \quad —⑥$$

Here, $\epsilon = K\epsilon_0$

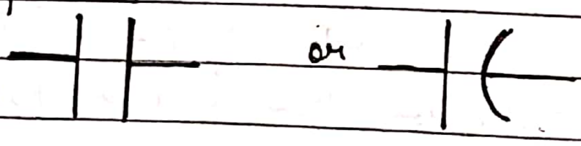
$$\therefore \quad C' = \frac{K\epsilon_0 A}{d} \quad —⑦$$

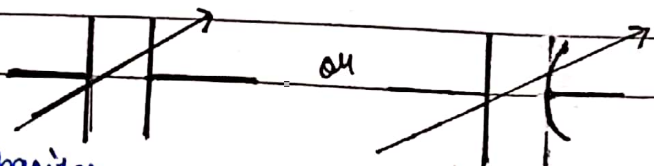
or $\quad \boxed{C' = KC}$

as $K > 1 \quad \therefore \quad \boxed{C' > C}$

Symbol of Capacitor

Fixed value capacitor





Variable value capacitor

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Q ⇒ There is an air filled 1 pico farad parallel plate capacitor. When the plate separation is doubled and the space is filled with the wax, the capacitance becomes 2 pico farad. Find the dielectric constant of wax.

$C_{air} = 1\,pF$ $\qquad$ $C_{wax} = 2\,pF$

$\frac{\epsilon_0 A}{d} = 1$ — ① $\qquad$ $\frac{K\epsilon_0 A}{2d} = 2$ — ②

Eqⁿ ② ÷ Eqⁿ ① $\qquad \frac{K}{2} = 2 \quad \therefore \boxed{K = 4}$

Q ⇒ The area of each plate of parallel plate capacitor is 100 cm² and the intensity of electric field b/w the plates is 100 N/C. Find the charge on each plate.

$A = 100\,cm^2 = 100 \times 10^{-4}\,m^2$

$E = 100\,N/C$

$E = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$

$q = EA\epsilon_0$

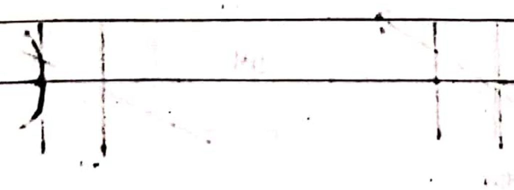
$q = 100 \times 100 \times 10^{-4} \times 8.85 \times 10^{-12}$

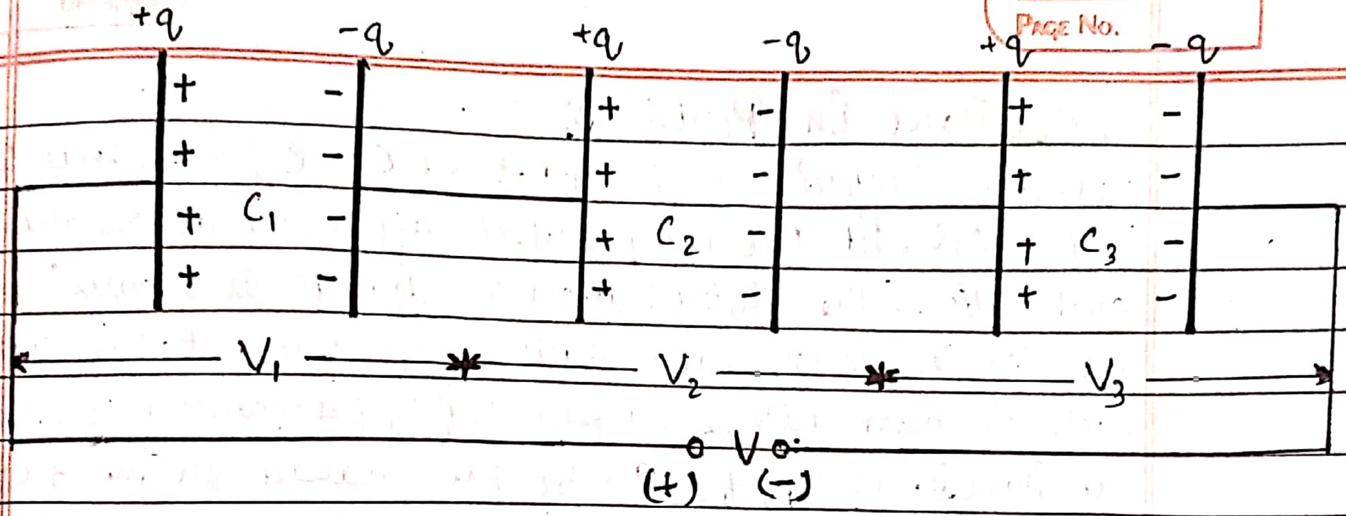
$q = 8.85 \times 10^{-12}\,C$

GROUPING OF CAPACITORS:

(A) Capacitors in Series

Consider 3 capacitors of capacitances C_1, C_2 and C_3 are connected in series. Let V is the potential difference across the combination. The distribution of charge is shown in the fig. Clearly the charge on each capacitor is same but potential difference across each of them will be different. Let V_1, V_2, V_3 be the potential difference across the 3 capacitors.





Then, $V_1 = \frac{q}{C_1}$ $V_2 = \frac{q}{C_2}$ $V_3 = \frac{q}{C_3}$.

As total potential difference is V

$$\therefore \quad V = V_1 + V_2 + V_3$$

Thus,
$$V = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \quad \text{--- (1)}$$

If C_s is the equivalent capacitance of the series combination, then on applying a potential V, it will also store the same amount of charge.

Thus,
$$V = \frac{q}{C_s} \quad \text{--- (2)}$$

from (1) & (2), we get

$$\frac{q}{C_s} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

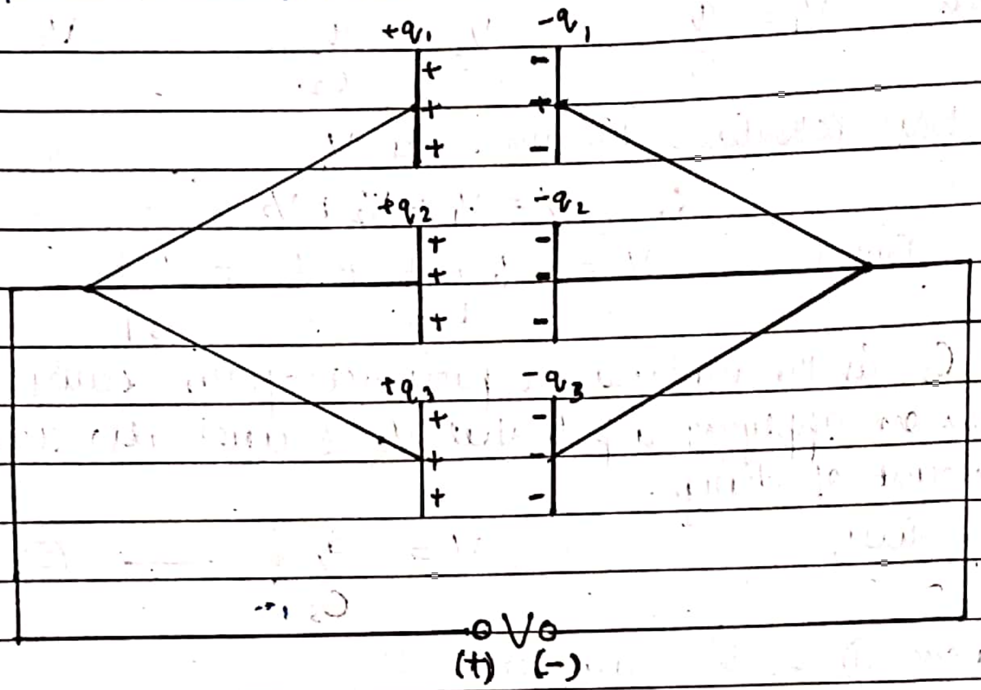
$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \quad (3)$$

In general :→ for a series combination of n capacitors.

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\cdots\cdots \frac{1}{C_n}} \quad (4)$$

Capacitors in Parallel

Consider 3 capacitors of capacitance C_1, C_2 & C_3 are connected in parallel. Let V be the potential difference across the combination. The distribution of charge is shown in the fig. The voltage across each capacitor is same but the charge on each of them will be different (in proportion of their capacitances). Let q_1, q_2, q_3 be the charges on the 3 capacitors.



$q_1 = C_1 V$ $\quad$ $q_2 = C_2 V$ $\quad$ $q_3 = C_3 V$

If q is the total charge, then

$$q = q_1 + q_2 + q_3$$
$$q = C_1 V + C_2 V + C_3 V$$
$$q = (C_1 + C_2 + C_3) V \quad \text{--- (1)}$$

If C_P is the equivalent capacitance, then,

$$q = C_P V \quad \text{--- (2)}$$

from (1) & (2)

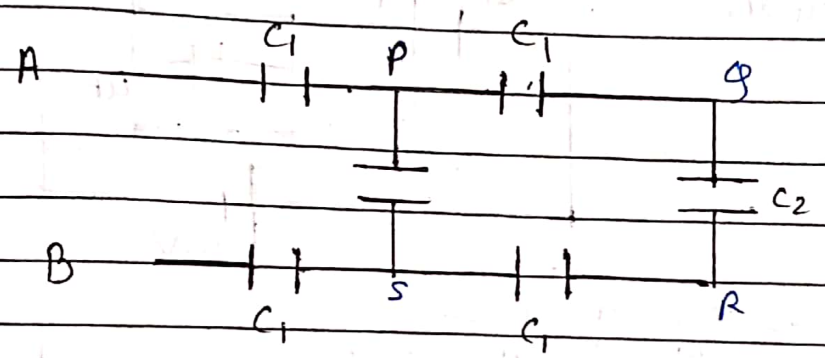
$$C_P \cancel{V} = (C_1 + C_2 + C_3) \cancel{V}$$

$$\boxed{C_P = C_1 + C_2 + C_3} \quad \text{--- (3)}$$

In General → for n capacitors

$$\boxed{C_P = C_1 + C_2 + C_3 + \text{- - - - - - - - -} + C_n} \quad \text{--- (4)}$$

Q:) In the fig. shown below calculate the equivalent capacitance of the networks b/w the points A and B. $C_1 = 3$ pF & $C_2 = 2$ pF

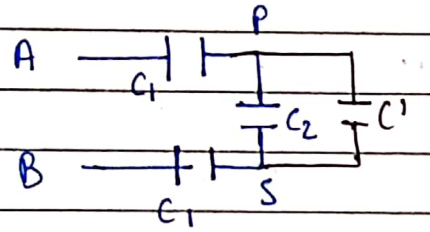


along the path PQRS the three capacitors are in series (C_1, C_2 & C_3)

∴ equivalent capacitance of these three (say C')

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{3} + \frac{1}{2} + \frac{1}{3} = \frac{7}{6}$$

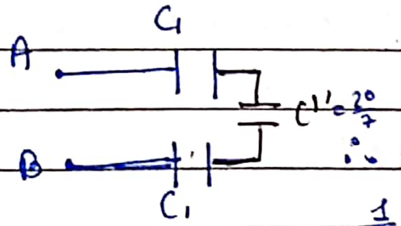
$$\therefore C' = \frac{6}{7} \text{ pF}$$



Now, C_1 & C' are in parallel

∴ eq. capacitance of these two (say C'') is given by -

$$C'' = C_2 + C' = 2 + \frac{6}{7} \Rightarrow C'' = \frac{20}{7} \text{ pF}$$



Now C_1, C'' & C_1 are in series

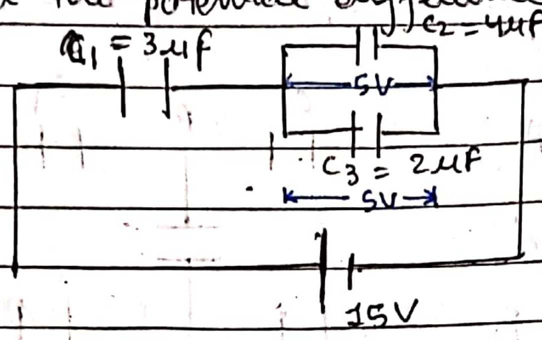
∴ eq. capacitance C is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C''} + \frac{1}{C_1} = \frac{1}{3} + \frac{1}{20} + \frac{1}{3}$$

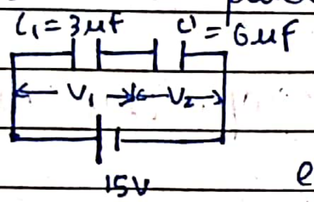
$$\frac{1}{C} = \frac{21 + 21 + 20}{60} = \frac{61}{60}$$

$$C = \frac{60}{61} \text{ pF} \approx 1 \text{ pF}$$

Q⇒ Find the charges on different capacitors shown in the fig. below. Also find the potential difference across each capacitor.



C_2 & C_3 are in parallel



$\therefore C' = C_2 + C_3 = 4\mu f + 2\mu f$
$= 6\mu f$

eq. capacitance of C_1 & C' (say C)

$$C = \frac{6 \times 3}{6+3} = 2\mu f$$

charge in each capacitance

$$q = CV = 2 \times 15 = 30\mu C$$

Now, $V_1 = \frac{q}{C_1} = \frac{30}{3} = 10V$

$$V_2 = \frac{q}{C'} = \frac{30}{6} = 5V$$

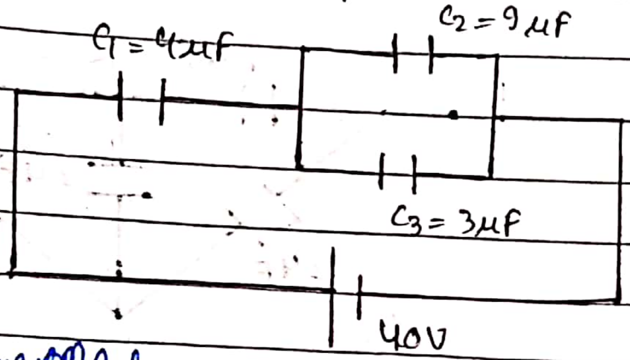
as C_2 & C_3 are in parallel
∴ Voltage across each of them will be same

$$q_1 = C_1 V_1$$
$$q_1 = 3 \times 10 = 30\mu C$$

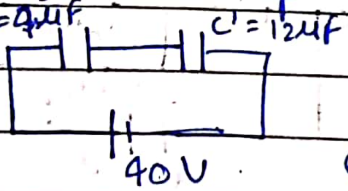
$$q_2 = C_2 \times V_2$$
$$q_2 = 4 \times 5 = 20\mu C$$

$$q_3 = C_3 V_2$$
$$q_3 = 2 \times 5 = 10\mu C$$

Q=) Find the charge on different capacitors shown in the fig. below.



C_2 & C_3 are in parallel



$\therefore C'_x = C_2 + C_3 = (9+3)\,\mu F = 12\,\mu F$

eq. capacitance of C_1 & C' (say C)

$$C = \frac{C_1 \times C'}{C_1 + C'} = \frac{4 \times 12}{4+12} = \frac{4 \times 12^3}{16_4} = 3\mu F$$

charge in each capacitor (C_1 & C')

$$q = CV = 3 \times 40 = 120\,\mu F$$

Now, $V_1 = \frac{q}{C_1} = \frac{120}{4} = 30\,V$

$V_2 = \frac{q}{C'} = \frac{120}{12} = 10\,V$

as C_2 & C_3 are in parallel

$\therefore$ Voltage across each of them will be same

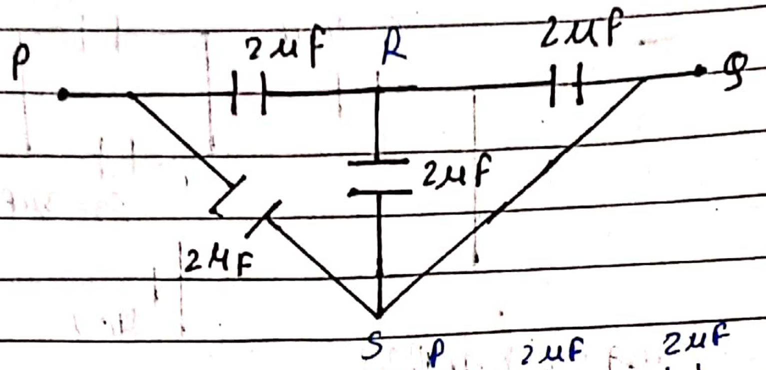
$q_1 = C_1 V_1$

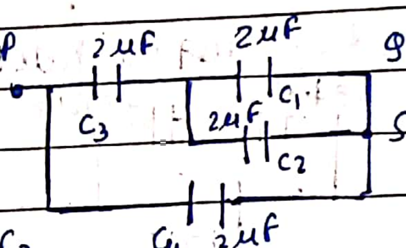
$q_1 = 4 \times 30 = 120\,\mu C$

$q_2 = C_2 \times V_2 = 9 \times 10 = 90\,\mu C$

$q_3 = C_3 \times V_2 = 3 \times 10 = 30\,\mu C$

Q⇒ Find the equivalent capacitance b/w P & Q.



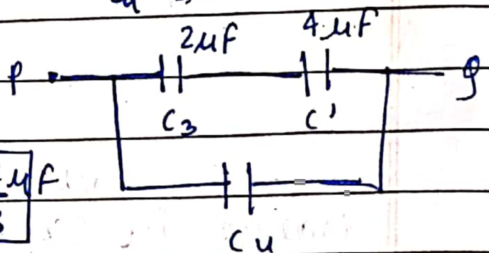


C_1 & C_2 are in parallel,

$C' = C_1 + C_2 = 4\mu F$

⇒ their equivalent in series with C_3

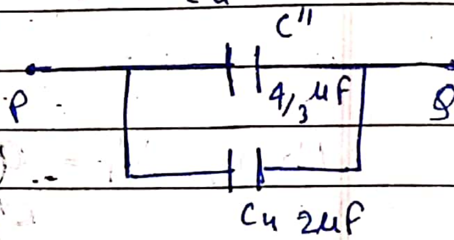
∴ $\frac{1}{C''} = \frac{1}{C_3} + \frac{1}{C'}$



$\frac{1}{C''} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4} \Rightarrow \boxed{C'' = \frac{4}{3}\mu F}$

(C'' & C_4)

Now, the 2 capacitors are in parallel



∴ $C_{eq} = \frac{4}{3} + 2 = \frac{10}{3}\mu F$ — Ans

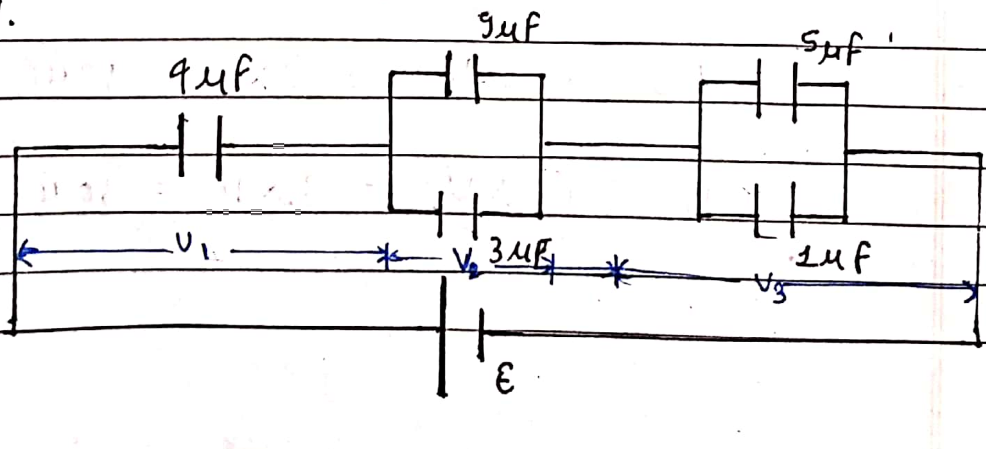
Q⇒ In the fig shown, find :-

(i) Potential difference across each capacitor.

(ii) e m f of the battery.

(iii) charge stored on each capacitor

Given that the potential difference across the capacitor of 3μF is 10 V.

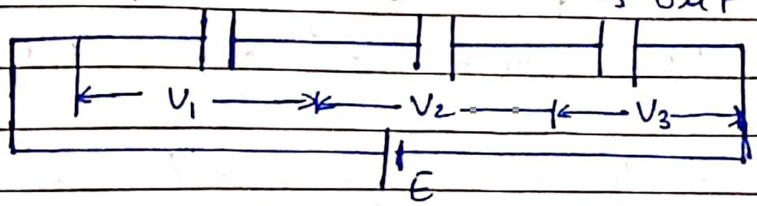


Solⁿ:- (i) as $9\mu F$ is in parallel with $3\mu F$

$\therefore$ Potential across $9\mu F = 10V$

eq. of $9\mu F$ and $3\mu F = 12\mu F$

$5\mu F$ & $1\mu F$ are also in $\parallel$el $\therefore$ eq. of $5\mu F$ & $1\mu F = 6\mu F$



as $q = CV \Rightarrow V = \frac{q}{C}$

If q is constant, then $V \propto \frac{1}{C}$

In 2nd fig all capacitors are in series

$\therefore$ q is same. $\Rightarrow V \propto \frac{1}{C}$

as $C_1 = \frac{C_2}{3}$ $\therefore$ $\boxed{V_1 = 3V_2 = 30V}$

and $C_3 = \frac{C_2}{2}$ $\therefore$ $\boxed{V_3 = 2V_2 = 20V}$

(ii) Charge on each capacitor

(a) $4\mu F \Rightarrow q = CV \Rightarrow q = 4 \times V_1 = 4 \times 30 = 120\mu C$

(b) $9\mu F \Rightarrow q = 9 \times 10 = 90\mu C$

(c) $3\mu F \Rightarrow q = 3 \times 10 = 30\mu C$

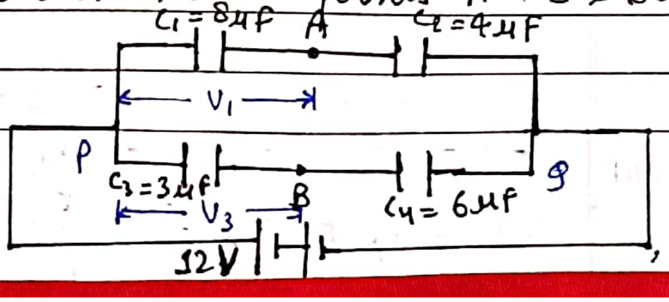
(d) $5\mu F \Rightarrow q = 5 \times 20 = 100\mu C$

(e) $1\mu F \Rightarrow q = 1 \times 20 = 20\mu C$

(iii) emf of battery $= V_1 + V_2 + V_3$

$= 30 + 10 + 20 = 60V$

Q:- Find the potential difference b/w the points A & B shown in the circuit below:-



eq. capacitance of upper branch $\Rightarrow \frac{8\times4}{8+4} = \frac{32}{12} = \frac{8}{3}\mu F$

eq. capacitance of lower branch $\Rightarrow \frac{6\times3}{6+3} = \frac{18}{9} = 2\mu F$

$q_1 = q_2 = C_{up} \times 12 = \frac{8}{3} \times 12 = 32\mu C$

$q_3 = q_4 = C_{low} \times 12 = 2 \times 12 = 24\mu C$

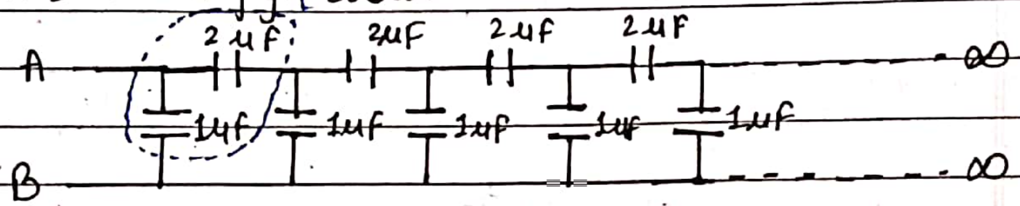
Now,

$$V_1 = \frac{q_1}{C_1} = \frac{32\mu C}{8\mu F} = 4V$$

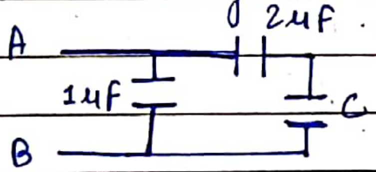
$$V_3 = \frac{q_3}{C_3} = \frac{24}{3} = 8V$$

$\therefore$ Pot. diff b/w A & B i.e. $V_A - V_B = 8-4 = 4V$

Q $\Rightarrow$ Find the capacitance of the infinite ladder b/w the points A and B in the fig. shown.



Let the equivalent capacitance of the given network be C.
As the circuit consists infinite steps.
$\therefore$ on adding or removing one step (consisting 1μF & 2μF)



Now clearly 2μF (C_1) & C are in series and their resultant in parallel with C_2, which gives eq. capacitance (C).

$$\therefore C_{eq} = C = \frac{CC_1}{C+C_1} + C_2$$

$$C = \frac{2C}{2+C} + 1$$

$$2C + C^2 = 2C + 2 + C \Rightarrow 2C + C^2 = 3C + 2$$

$C^2 - C - 2 = 0 \Rightarrow C^2 - (2-1)C - 2 = 0$

$C^2 - 2C + C - 2 = 0$

$C(C-2) + 1(C-2) = 0$

$\boxed{C-2=0}$, $\boxed{C=2}$ or $C+1=0$ ✗ (as C can't be -ve)

$\therefore \boxed{C_{eq} = 2\mu F}$

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ENERGY OF A CHARGED CAPACITOR

"The work done in charging a capacitor is stored in the capacitor in the form of electric potential energy."

This energy is supplied by the battery at the expense of its stored chemical energy and can be recovered by allowing the capacitor to discharge.

Expression for the energy stored in a capacitor:-

Consider a capacitor of capacitance C. Let on applying a charge q, to it through a battery, it acquires a potential V.

Then $$q = CV \quad \text{--- (1)}$$

Small work done by the battery to supply an infinitesimally small charge dq, to the capacitor, ∃

$$dW = V\,dq = (q/C)\,dq \quad [\text{as } V = q/C] \quad \text{--- (2)}$$

∴ Total amount of work done by battery to supply a charge q, to the capacitor.

$$W = \int_0^q \frac{q}{C}\,dq$$

$$W = \frac{1}{C}\int_0^q q\,dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$W = \frac{1}{2}\frac{q^2}{C} \quad \text{--- (3)}$$

This work is stored in the form of electric potential energy therefore electric potential energy of the capacitor.

$$\boxed{U = \frac{1}{2}\frac{q^2}{C}} \quad \text{--- (4)}$$

as $q = CV$
∴ from eqⁿ (4)

$$\boxed{U = \frac{1}{2}CV^2} \quad \text{--- (5)}$$

Further, as $C = \frac{q}{V}$

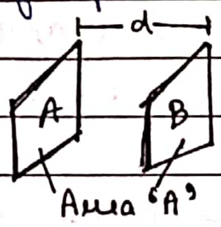
$\therefore$ eqn (5) gives $\boxed{U = \frac{1}{2} qV}$ — (6)

ENERGY DENSITY OF A CHARGED CAPACITOR

Energy density of a charged capacitor = energy stored per unit volume of capacitor

$$= \frac{\text{Energy stored in capacitor}}{\text{Volume of capacitor}}$$

$$\bar{U} = \frac{\frac{1}{2} qV}{Ad}$$



$$\bar{U} = \frac{1}{2} \frac{q}{A} \frac{V}{d}$$

$$\bar{U} = \frac{1}{2} \sigma E \quad — (1)$$

as $E = \frac{\sigma}{\epsilon_0}$ $\quad \therefore \sigma = \epsilon_0 E$

$\therefore$ from (1) $\quad \bar{U} = \frac{1}{2} (\epsilon_0 E) E$

$$\boxed{\bar{U} = \frac{1}{2} \epsilon_0 E^2} \quad — (2)$$

Thus, an electric field E can be regarded as a seat of energy with energy density equal to $\frac{1}{2} \epsilon_0 E^2$.

REDISTRIBUTION OF CHARGES : Common Potential

Consider two capacitors A & B of capacitance C_1 & C_2 respectively are charged to the potentials V_1 & V_2 respectively. Let q_1 & q_2 be the charges and U_1 & U_2 be the energies stored on each of them. $\therefore$ $q_1 = C_1 V_1$ — (1)

$q_2 = C_2 V_2$ — (2)

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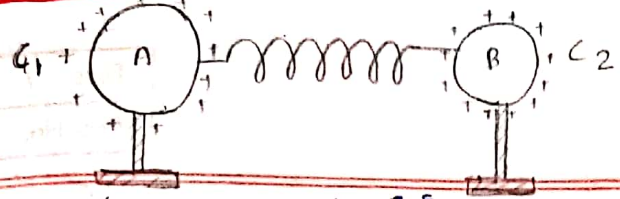

$$U_1 = \frac{1}{2} C_1 V_1^2 \quad \text{——— (3)}$$
$$U_2 = \frac{1}{2} C_2 V_2^2 \quad \text{——— (4)}$$

Now, if they are joined in parallel, the charge begins to flow from a capacitor at higher potential to that at lower potential till their potentials become equal i.e., the charges on them are redistributed being the total charge on the system is constant which is $q_1 + q_2$

If V is the common potential, then

$$V = \frac{\text{total charge}}{\text{combined capacitance}} = \frac{q_1 + q_2}{C_1 + C_2}$$

$$\boxed{V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}} \quad \text{——— (5)}$$

Suppose after redistribution the charge on A is q_1' & on B is q_2', then

$$\frac{q_1'}{q_2'} = \frac{C_1 V}{C_2 V} = \frac{C_1}{C_2} \quad \text{——— (6)}$$

Thus, on connecting the two charged capacitors, the redistributed charges are in the ratio of their capacitance.

Quantity of the Transferred Charge

Let the charge is transferred from A to B.

The charge transferred from A to B is $q_1 - q_1'$

$$q_1 - q_1' = C_1 V_1 - C_1 V$$
$$= C_1 (V_1 - V)$$
$$q_1 - q_1' = C_1 \left[V_1 - \frac{(C_1 V_1 + C_2 V_2)}{C_1 + C_2} \right]$$

$$\boxed{q_1 - q_1' = \frac{C_1 C_2 (V_1 - V_2)}{C_1 + C_2}} \quad \text{——— (7)}$$

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LOSS OF ENERGY IN DISTRIBUTION OF CHARGES

Before connecting the total energy of the capacitor is

$$U = U_1 + U_2$$

$$U = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$\boxed{U = \frac{1}{2}\left(C_1V_1^2 + C_2V_2^2\right)} \quad \text{—— (8)}$$

After connecting the total energy of the combination becomes

$$U' = \frac{1}{2} \times \text{combined capacitance} \times (\text{common potential})^2$$

$$\boxed{U' = \frac{1}{2}(C_1+C_2)V^2} \quad \text{—— (9)}$$

from eq^n (5) & eq^n (9) we get

$$U' = \frac{1}{2}(C_1+C_2)\left[\frac{C_1V_1 + C_2V_2}{C_1+C_2}\right]^2$$

$$\boxed{U' = \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1+C_2}} \quad \text{—— (10)}$$

$\therefore$

$$U - U' = \frac{1}{2}(C_1V_1^2 + C_2V_2^2) - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{C_1+C_2}$$

$$U - U' = \frac{1}{2}\left[\frac{(C_1V_1^2 + C_2V_2^2)(C_1+C_2) - (C_1^2V_1^2 + C_2^2V_2^2 + 2C_1C_2V_1V_2)}{C_1+C_2}\right]$$

$$= \frac{1}{2}\left[\frac{\cancel{C_1^2V_1^2} + \cancel{C_2^2V_2^2} + C_1C_2V_1^2 + C_2C_1V_2^2 - \cancel{C_1^2V_1^2} - \cancel{C_2^2V_2^2} - 2C_1C_2V_1V_2}{C_1+C_2}\right]$$

$$= \frac{1}{2}\left[\frac{C_1C_2\,(V_1^2 + V_2^2 - 2V_1V_2)}{C_1+C_2}\right]$$

$$\boxed{U - U' = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2} \quad \text{—— (11)}$$

As the RHS of the above eq^n is a +ve quantity, therefore $U' < U$, hence in this process some energy is lost in the form of heat & electromagnetic radiation.

Q:) A $10\mu F$ capacitor is charged by 30V d.c. supply and then connected across an uncharged $50\mu F$. Calculate (a) the final potential difference across the combination and (b) the initial & final energies. How will you account for the difference in energy.

Sol^n $C_1 = 10\mu F$ $\qquad$ $C_2 = 50\mu F$

$V_1 = 30V$ $\qquad$ $V_2 = 0V$

(a) Common Potential $= V = \dfrac{C_1V_1 + C_2V_2}{C_1+C_2}$

$$= \frac{(10\times 30) + 0}{10+50} = \frac{300}{60} = 5V$$

(b) $U_i = U_1 + U_2$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$$

$$= \frac{1}{2}\left[10\times10^{-6}\times(30)^2 + 50\times10^{-6}\times(0)^2\right]$$

$$= \frac{1}{2}\times 900\times 10^5 \times 10^{-6} = 45\times10^{-4} J$$

$$U_f = \frac{1}{2}\left[(C_1+C_2)V^2\right]$$

$$= \frac{1}{2}(10\times10^{-6} + 50\times10^{-6})(5)^2$$

$$= \frac{1}{2}\times 60\times10^{-6}\times 25 = 750\times10^{-6}$$

$$U_f = 7.5\times10^{-4} J$$

$U_f < U_i$ $\qquad$ $U_i - U_f = 37.5\times10^{-4} J$

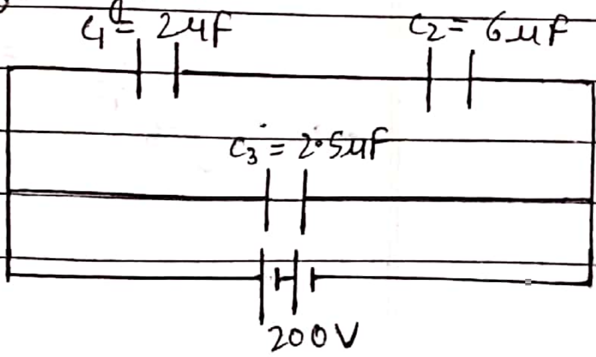
Hence, some energy $(37.5\times10^{-4} J)$ is lost in the form of heat and electromagnetic radiation.

Q⇒ In the fig. shown below find :-)

(i) Total capacitance, charge & energy of the system.

(ii) Charges on seperate capacitors.

(iii) Energy stored on each capacitor.



(i) C_{eq} = C_1 & C_2 are in series & their equivalent is in ∥ with C_3

$$\therefore C_{eq} = \frac{6 \times 2}{6+2} + 2.5 = \frac{12}{8} + 2.5$$

$$C_{eq} = 4\,\mu F$$

$\therefore$ charge $q = C_{eq} \times V = 4 \times 200 = 800\,\mu C$

$$U = \frac{1}{2} C_{eq} V^2 = \frac{1}{2}(4 \times 10^{-6}) \times (200)^2$$

$$= 2 \times 4 \times 10^{-2}$$

$$= 0.08\,J$$

(ii) $V_1 + V_2 = 200$ —— ①

as $V \propto \frac{1}{C}$ (for q be same)

$$\frac{V_1}{V_2} = \frac{C_2}{C_1} = \frac{6}{2} = 3 \text{ —— ②}$$

$$V_1 = 3V_2$$

$$\therefore 3V_2 + V_2 = 200$$

$$V_2 = 50\,V \quad \& \quad V_1 = 150\,V$$

$$q_1 = C_1 V_1 = 2 \times 150 = 300\,\mu C$$

$$q_2 = C_2 V_2 = 6 \times 150 = 300\,\mu C$$

$$q_3 = C_3 V = 2.5 \times 200 = 500\,\mu C$$

(iii) $U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(2 \times 10^{-6})(150)^2$

$$= 225 \times 10^{-4}$$

$$= \cancel{0.225}\; 0.0225\,J$$

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(6 \times 10^{-6})(50)^2$$

$$= 15 \times 10^{-4}$$

$$= 0.0075\,J$$

$$U_3 = \frac{1}{2} C_3 V^2 = \frac{1}{2} \times (2.5 \times 10^{-6})(200)^2$$

$$= 5 \times 10^{-6} \times 10^4$$

$$= 0.05\,J$$

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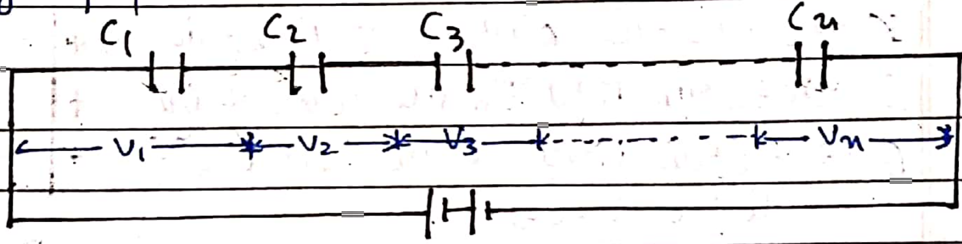
NOTE:→ In either combination (series/parallel) of the capacitors. The total energy stored in the combination is equal to the sum of the energy stored of individual capacitors.

Proof:→ (1) **For Series Combination:**→ consider 'n' capacitors of capacitance $C_1, C_2, C_3 ------ C_n$ are connected in series to a battery of potential V.



$$U_{comb.} = \frac{1}{2}\frac{q^2}{C_{eq}}$$

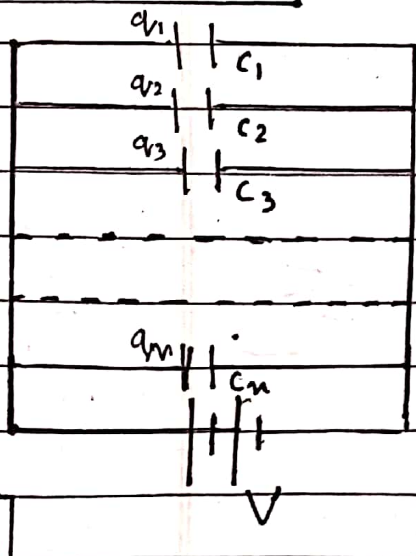
$$U_{comb} = \frac{1}{2}q^2\left[\frac{1}{C_{eq}}\right]$$

$$U_{comb} = \frac{1}{2}q^2\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} + ---------- \frac{1}{C_n}\right]$$

$$U_{comb} = \frac{1}{2}\frac{q^2}{C_1} + \frac{1}{2}\frac{q^2}{C_2} + \frac{1}{2}\frac{q^2}{C_3} + ------------ + \frac{1}{2}\frac{q^2}{C_n}$$

i.e. $\boxed{U_{comb} = U_1 + U_2 + U_3 + ---------- + U_n}$

(2) **For Parallel Combination:**→



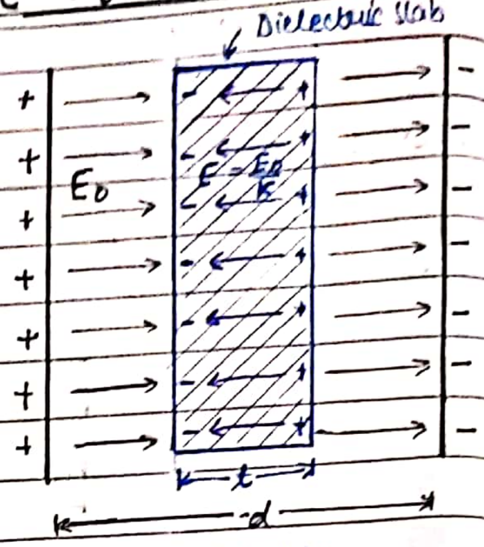
$$U_{comb} = \frac{1}{2}C_{eq}V^2$$

$$U_{comb} = \frac{1}{2}[C_1 + C_2 + C_3 + ------ C_n]V^2$$

$$U_{comb} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 + ---------- + \frac{1}{2}C_nV^2$$

$$\boxed{U_{comb} = U_1 + U_2 + U_3 + ---------- + U_n}$$

Imp Capacitance of a Parallel Plate Capacitor with partially filled dielectric slab b/w its plates



Let $A \rightarrow$ area of each plate of capacitor
$d \rightarrow$ distance b/w the two plates
$t \rightarrow$ thickness of the dielectric slab
$(t < d)$
$K \rightarrow$ dielectric constant of dielectric
$E_0 \rightarrow$ Electric Field in the region of air $(d-t)$
$E \rightarrow$ Electric Field in the region of dielectric. (t)
$C_0 \rightarrow$ Capacitance of air/vaccum filled parallel plate capacitor.
$C \rightarrow$ capacitance of partially filled dielectric parallel plate capacitor.

When there is no dielectric then
Clearly, $C_0 = \frac{\epsilon_0 A}{d}$ —— ①

after placing the dielectric, if the total potential is V across the capacitor plates, then,

$$V = V_{air} + V_{dielectric} \quad \text{—— ②}$$

$$V = [E_0(d-t)] + [Et]$$

$$V = E_0(d-t) + \frac{E_0}{K}t \qquad \left[\text{as } \frac{E_0}{E} = K\right]$$

$$V = E_0\left(d - t + \frac{t}{K}\right)$$

$$V = \frac{\sigma}{\epsilon_0}\left(d - t + \frac{t}{K}\right) \qquad \left[\because E_0 = \frac{\sigma}{\epsilon_0}\right]$$

$$\boxed{V = \frac{q}{A\epsilon_0}\left(d - t + \frac{t}{K}\right)} \quad \text{—— ③} \qquad \left[\because \sigma = \frac{q}{A}\right]$$

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If q is the charge on capacitor plates

then $C = \frac{q}{V} = \frac{q}{\frac{q}{A\epsilon_0}\left(d - t + \frac{t}{K}\right)}$

$$\boxed{C = \frac{\epsilon_0 A}{\left(d - t + t/K\right)}} \quad \text{—— (4)}$$

⇒ If space b/w the plates is filled completely with dielectric then, $t = d$

∴ from eq^n (4) $C' = \frac{\epsilon_0 A}{d - d + d/K}$

$C' = \frac{\epsilon_0 A}{d/K}$

$$\boxed{C' = K\frac{\epsilon_0 A}{d}} \quad \text{or} \quad \boxed{C' = K C_0} \quad \text{—— (5)}$$

✶ **SPECIAL CASE** :→ If in place of dielectric slab, a conducting slab is placed (partially)

then, from eq^n (2)

$V = V_{air} + V_{conductor}$

$V = [E_0(d-t)] + Et$ —— (6)

but as inside the conductor

$E = 0$

∴ from eq^n (6) $V = E_0(d-t)$

$V = \frac{\sigma}{\epsilon_0}(d-t)$

$V = \frac{q}{A\epsilon_0}(d-t)$ —— (7)

as $C = \frac{q}{V}$ ∴ from eq^n (7)

$C = \frac{q}{\frac{q}{A\epsilon_0}(d-t)}$

$$\boxed{C = \frac{\epsilon_0 A}{d-t}} \text{ (8)} \quad \text{or} \quad C = \frac{\epsilon_0 A}{d} \cdot \frac{d}{(d-t)} \quad \text{or} \quad \boxed{C = \left(\frac{d}{d-t}\right) \cdot C_0} \text{ (9)}$$

or from eq^n (4) $\quad C = \dfrac{\epsilon_0 A}{d - t + t/K}$

for conductor $K = \infty$

$$\therefore \boxed{C = \frac{\epsilon_0 A}{d - t}}$$

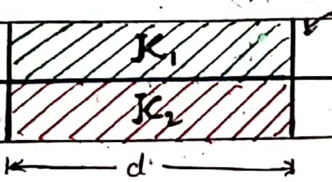
$\Rightarrow$ If conducting slab fills the whole space b/w the plates.

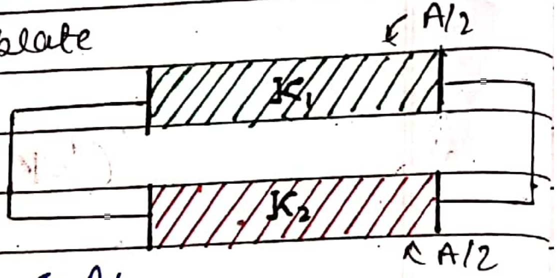
Then, $t = d$

$\therefore$ from eq^n (8) $\quad \boxed{C = \infty}$ — (10)

Q$\Rightarrow$ Find the capacitance in each case:$\rightarrow$

(I)





$C_1 = K_1 \dfrac{\epsilon_0 A/2}{d} \qquad C_2 = K_2 \dfrac{\epsilon_0 A/2}{d}$

$C_1 = \dfrac{K_1 \epsilon_0 A}{2d} \qquad C_2 = K_2 \dfrac{\epsilon_0 A}{2d}$

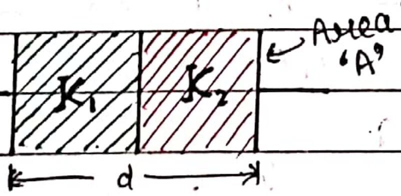
as C_1 and C_2 are parallel

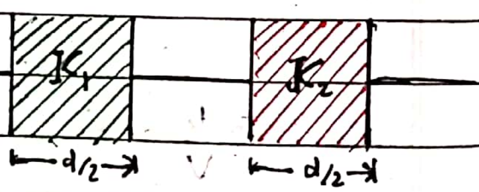
$$\therefore \boxed{C_{eq} = C_1 + C_2 = \frac{\epsilon_0 A}{2d}(K_1 + K_2)}$$

In general for parallel type arrangement of n capacitors, when $A_1 = A_2 = A_3 = \text{-----} = A_n = A/n$

$$\boxed{C_{eq} = \frac{\epsilon_0 A}{nd}(K_1 + K_2 + K_3 + \text{-------} + K_n)}$$

(II)





$C_1 = K_1 \dfrac{\epsilon_0 A}{d/2} \qquad C_2 = K_2 \dfrac{\epsilon_0 A}{d/2}$

$C_1 = 2K_1 \dfrac{\epsilon_0 A}{d} \qquad C_2 = 2K_2 \dfrac{\epsilon_0 A}{d}$

as C_1 and C_2 are in series

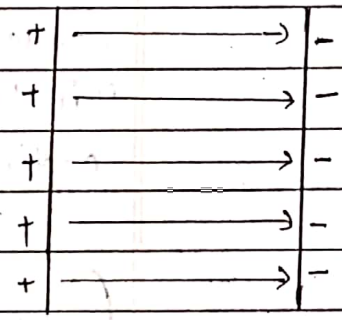
$$\therefore \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$C_{eq} = \frac{\frac{2K_1 \epsilon_0 A}{d} \times \frac{2K_2 \epsilon_0 A}{d}}{\frac{2\epsilon_0 A}{d}(K_1 + K_2)}$$

$$\boxed{C_{eq} = \frac{2\epsilon_0 A}{d}\left[\frac{K_1 K_2}{K_1 + K_2}\right]}$$

Force Between the Plates of a Capacitor



Electric field due to plate carrying charge q

$$E' = \frac{\sigma}{2\epsilon_0} = \frac{q}{2A\epsilon_0} \quad \text{--- (1)}$$

Now, if another plate is placed near it then force on it.

$$F = qE' = q\left(\frac{q}{2A\epsilon_0}\right)$$

$$\boxed{F = \frac{q^2}{2A\epsilon_0}} \quad \text{--- (2)}$$

$$F = \frac{1}{2} \times q\left(\frac{q}{A\epsilon_0}\right)$$

$$F = \frac{1}{2} q\left(\frac{\sigma}{\epsilon_0}\right)$$

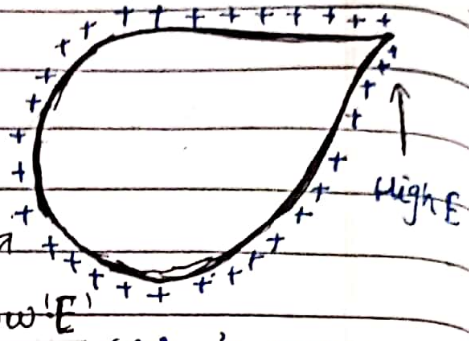
$$\boxed{F = \frac{1}{2} qE} \quad \text{--- (3)}$$

E ⇒ Electric field b/w the capacitor plate

Discharge Action of Sharp Points : Corona Discharge

When a spherical conductor of radius r carries a charge q, its surface charge density is

$$\sigma = \frac{q}{A} = \frac{q}{4\pi r^2}$$



Electric field on the surface is

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 r^2}$$

as for pointed conductors 'r' & hence 'A' is very small $\therefore$ σ is large

$$\therefore \quad E = \frac{\sigma}{\epsilon_0} \quad \therefore E \text{ is large}$$

This causes the ionisation or electrical breakdown of the surrounding air.

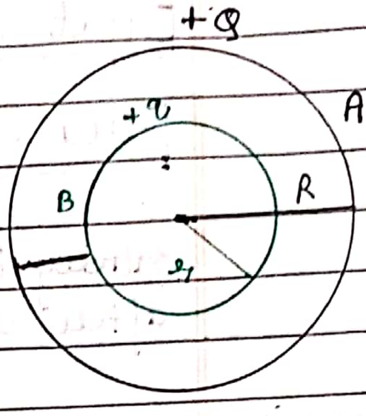
This process by which the charge at the pointed end of a conductor gets discharged is called corona discharge.

Collecting Action of a Hollow Conductor

If a small positively charged sphere is enclosed by a spherical shell, then on connecting the inner sphere ~~with~~ the shell through a wire, the ~~sph~~ charge supplied to the sphere would necessarily flow from the sphere to the shell irrespective of the fact how so ever large the potential of the shell may grow. The charge immediately shifts to the outer surface of the shell.

Consider A is a conducting shell of radius 'R', having a charge $+Q$. Therefore, potential inside and at the surface of the charged conducting shell 'A' is

$$V_1 = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \quad \text{---} (1)$$



Let a small conducting sphere B of radius 'r' and charge '+q' is placed at the centre of a shell, then

The potential due to sphere B at the surface of the shell 'A'

$$V_2 = K\frac{q}{R} \quad \text{---} (2)$$

and potential due to sphere B at its own surface ~~of the shell 'A'~~

$$V_3 = K\frac{q}{r} \quad \text{---} (3)$$

∴ Total potential at the surface of sphere 'A'

$$V_A = V_1 + V_2 = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{R} + \frac{q}{R}\right] \quad \text{---} (4)$$

Total potential at the surface of sphere B

$$V_B = V_3 + V_4$$

$$V_B = \frac{1}{4\pi\epsilon_0}\left[\frac{Q}{R} + \frac{q}{r}\right] \quad \text{---} (5)$$

Clearly $V_B > V_A$

∴ Potential Difference $V_B - V_A$

$$V = V_B - V_A = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right] \quad \text{---} (6)$$

independent of Q

Clearly as $V_B > V_A$, therefore when the sphere is connected to the shell by a wire, the charge supplied to the sphere will immediately flow from the sphere to the shell.

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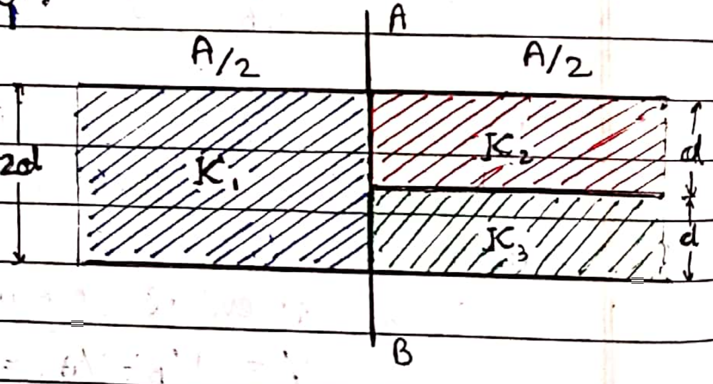
Effect of inserting a dielectric b/w the Plates of a Parallel Plate Capacitor

	Battery is disconnected after charging	Battery remain connected
(1) Capacitance (C)	increase $C' = KC$	increase $C' = KC$
(2) Charge (q)	same $q' = q$	increase $q' = Kq$
(3) Electric Field (E)	$E' = \frac{E}{K}$ (decrease)	$E' = E$ (same)
(4) Potential Difference (V)	$V' = \frac{V}{K}$ (decrease)	$V' = V$ (same)
(5) Energy (U)	$U = \frac{1}{2}\frac{q^2}{C}$ as q is const. & C increases $\therefore$ U decreases $U' = U/K$	$U = \frac{1}{2}qV$ as q incre. & V is const. $\therefore$ U increases $U' = KU$

Q ⇒ Find the equivalence capacitance b/w the points A & B in the fig. shown below.



Sol^n ⇒ Using $C = \frac{K\epsilon_0 A}{d}$

$$C_1 = K_1 \frac{\epsilon_0 A/2}{2d}$$

$$C_1 = \frac{K_1 \cdot \epsilon_0 A}{4d} \quad \text{—— (1)}$$

$$C_2 = K_2 \frac{\epsilon_0 A/2}{d}$$

$$C_2 = \frac{K_2 \epsilon_0 A}{2d} \quad \text{—— (2)}$$

$$C_3 = K_3 \frac{\epsilon_0 A/2}{d}$$

$$C_3 = \frac{K_3 \epsilon_0 A}{2d} \quad \text{—— (3)}$$

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as C_2 and C_3 are in series

$\therefore$ equivalent capacitance $C' = \dfrac{C_2 C_3}{C_2 + C_3}$ $\left[\text{as } \dfrac{1}{C'} = \dfrac{1}{C_2} + \dfrac{1}{C_3}\right]$

$$C' = \frac{\dfrac{K_2 \epsilon_0 A}{2d} \times \dfrac{K_3 \epsilon_0 A}{2d}}{\dfrac{\epsilon_0 A}{2d}(K_2 + K_3)}$$

$$C' = \frac{\epsilon_0 A}{2d} \frac{(K_2 K_3)}{(K_2 + K_3)}$$

Now C' and C_1 are in parallel

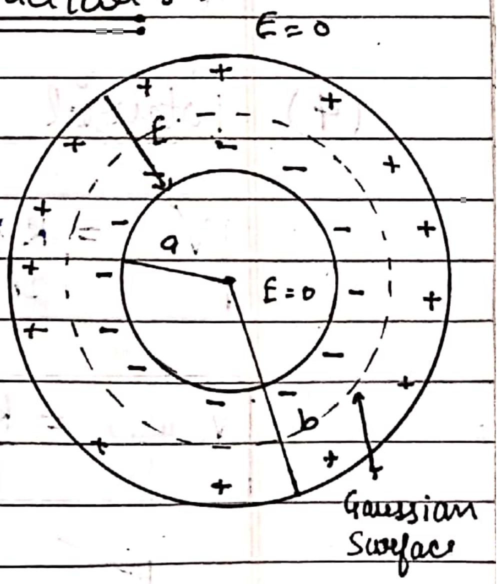
$\therefore$ equivalent capacitance $C = C_1 + C'$

$$C = \frac{K_1 \epsilon_0 A}{4d} + \frac{\epsilon_0 A}{2d} \frac{(K_2 K_3)}{(K_2 + K_3)}$$

$$\boxed{C = \frac{\epsilon_0 A}{2d}\left(\frac{K_1}{2} + \frac{K_2 K_3}{K_2 + K_3}\right)}$$

⇒ ADDITIONAL INFORMATION

(1) Capacitance of a Spherical Capacitor :-



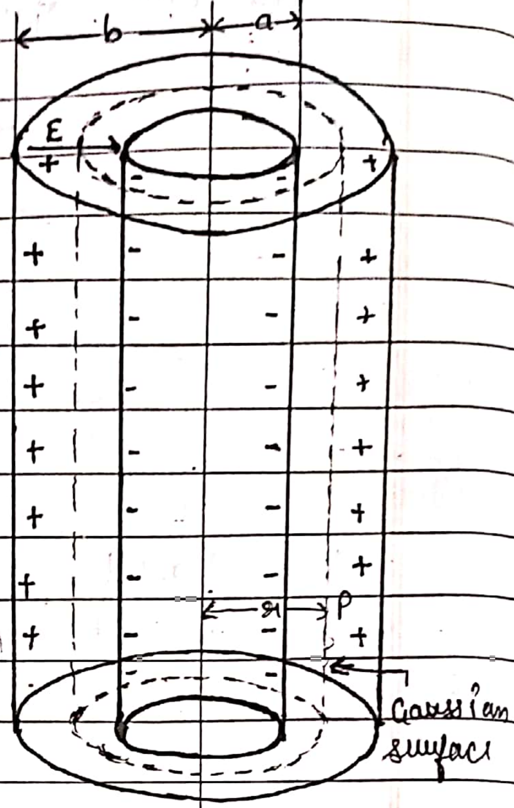
A spherical capacitor consists of two concentric spherical shells of inner radius 'a' and outer radius 'b'.

The capacitance of the spherical shell is

$$\boxed{C = 4\pi\epsilon_0 \left(\frac{ab}{a-b}\right)}$$

(2) Capacitance of a Cylindrical Capacitor :->

A cylindrical capacitor consists of two coaxial conducting cylinders of inner and outer radii a and b.



Capacitance of cylindrical capacitor is :->

$$\boxed{C = \frac{2\pi\epsilon_0}{\log_e\left(\frac{b}{a}\right)}}$$

(3) Electric Field of a Line Charge (Finite Length) :->

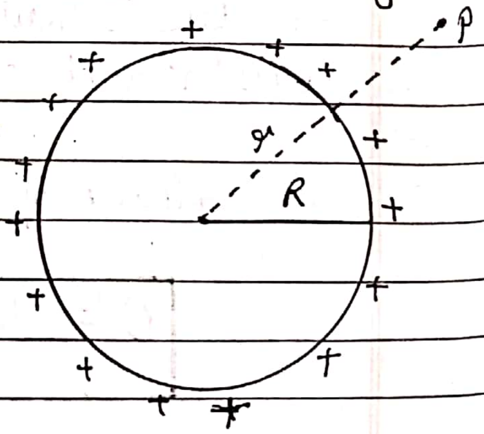
$$\boxed{E = \frac{1}{4\pi\epsilon_0}\frac{q}{x\sqrt{x^2+a^2}}}$$

where x is the distance of point of observation from line charge and length of the line charge is $2a$.

(4) Potential Due to Solid Sphere of the Charge :->

$$\boxed{V_{out} = \frac{kq}{r}}$$

R => radius of sphere.
r => dist. of observation point from the centre of sphere



$$\boxed{V_{surface} = \frac{kq}{R}}$$

$$\boxed{V_{inside} = \frac{kq}{R^3}\left[\frac{3}{2}R^2 - \frac{1}{2}r^3\right]}$$

(5) Potential on the axis of a Charged Ring :-

$$\boxed{V = \frac{kq}{\sqrt{R^2 + x^2}}}$$

R ⇒ radius of ring

x ⇒ dist. of observation point from centre of ring.

at centre ⇒ $x = 0$

$$\therefore \boxed{V_{centre} = \frac{kq}{R}}$$